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PLUME3D:

THREE-DIMENSIONAL PLUMES IN UNIFORM GROUND WATER FLOW

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16. ABSTRACT

A closed-form analytical solution for three-dimensional plumes was incorporated in an interactive computer program. The assumption of an infinite aquifer depth and uniform source mass rate and source location was overcome by using the principal of superposition in space and time. The source code was written in a subset of FORTRAN 77 and can be compiled with FORTRAN IV, FORTRAN 66 as well as FORTRAN 77. As a result, the code is nearly independent of hardware and operating system.

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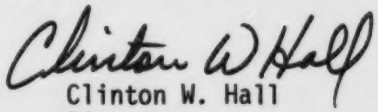
FOREWORD

EPA is charged by Congress to protect the Nation's land, air, and water systems. Under a mandate of national environmental laws focused on air and water quality, solid waste management and the control of toxic substances, pesticides, noise, and radiation, the Agency strives to formulate and implement actions which lead to a compatible balance between human activities and the ability of natural systems to support and nurture life.

The Robert S. Kerr Environmental Research Laboratory is the Agency's center of expertise for investigation of the soil and subsurface environment. Personnel at the Laboratory are responsible for management of research programs to: (a) determine the fate, transport and transformation rates of pollutants in the soil, the unsaturated zone and the saturated zones of the subsurface environment; (b) define the processes to be used in characterizing the soil and subsurface environment as a receptor of pollutants; (c) develop techniques for predicting the effect of pollutants on ground water, soil and indigenous organisms; and (d) define and demonstrate the applicability and limitations of using natural processes, indigenous to the soil and subsurface environment, for the protection of this resource.

This project was initiated to develop an interactive computer model which could be utilized to predict toxic chemical fate in homogeneous aquifers. The model should be useful in making comparisons, between chemicals, for idealized homogeneous aquifers. This model is not intended for addressing site specific problems where there is significant heterogeneity in the aquifer.



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ABSTRACT

A closed-form analytical solution for two dimensional plumes was incorporated in an interactive computer program. The assumption of an infinite aquifer depth and uniform source mass rate and source location was overcome by using the principal of superposition in space and time. The source code was written in a subset of FORTRAN 77 and can be compiled with FORTRAN IV, FORTRAN 66 as well as FORTRAN 77. As a result, the code is nearly independent of hardware and operating system.

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INTRODUCTION

This document describes a mathematical model and the associated computer program which can be used to estimate concentration distributions in a leachate plume which emanates from a point source. The model includes both linear adsorption and first-order reactions.

The use of the computer program is fairly simple, but represents only one tool which can aid in the analysis and understanding of ground-water contamination problems. The user must select the appropriate tools for the problem at hand, based on a sound understanding of the principles of ground-water hydrology, the physical problem, and the assumptions and limitations of the mathematical model.

SECTION I

MATHEMATICAL DEVELOPMENT

The differential equation describing the conservation of mass of a component in a saturated, homogeneous aquifer with uniform, steady flow in the x-direction can be written as

$$R_d \frac{\partial C}{\partial t} + V^* \frac{\partial C}{\partial x} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - R_d \lambda C \qquad (1)$$

where

C =	component mass per unit volume of fluid phase	M/L^3
D_x =	dispersion coefficient in x-direction	L^2/t
D_y =	dispersion coefficient in y-direction	L^2/t
D_z =	dispersion coefficient in z-direction	L^2/t
R_d =	retardation coefficient	
V^* =	average interstitial velocity in x-direction	L/t
x,y,z =	rectangular coordinates	L
λ =	first-order decay constant	$1/t$

The retardation coefficient accounts for partitioning of the component between the fluid and solid phases using a linear adsorption isotherm, and is defined as

$$R_d = 1 + \frac{\rho_B}{\theta} K_d \qquad (2)$$

where

ρ_B = bulk density of the aquifer M/L^3

θ = effective porosity

K_d = distribution constant for a linear adsorption isotherm $\frac{M/M}{M/L^3}$

Analytical Solution

A closed-form analytical solution to Equation 1 can be obtained by making a change of variables. Let

$$\tau = t/R_d \tag{3}$$

and

$$X = x - V^*\tau \tag{4}$$

Then Equation 1 is transformed to

$$\frac{\partial C}{\partial \tau} = D_x \frac{\partial^2 C}{\partial X^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - R_d \lambda C \tag{5}$$

with boundary conditions

$$C(X,y,z,o) = 0 \tag{6a}$$

$$C(X,y,\pm\infty,\tau) = 0 \tag{6b}$$

$$C(X,\pm\infty,z,\tau) = 0 \tag{6c}$$

$$C(\pm\infty, y, z, \tau) = 0 \tag{6d}$$

Equation 5 is a special form of the analogous equations in heat conduction, for which solutions are given by Carslaw and Jaeger (1959). The solution for an instantaneous point source of strength M_o at the origin is

$$C_i = \frac{M_o}{8\theta\sqrt{\pi^3 t^3 D_x D_y D_z}} \exp\left(-\frac{x^2}{4D_x \tau} - \frac{y^2}{4D_y \tau} - \frac{z^2}{4D_z \tau} - R_d \lambda \tau\right) \tag{7}$$

In terms of the untransformed variables,

$$C_i = \frac{M_o}{8\theta\sqrt{\pi^3 t^3 D_x D_y D_z}} \exp\left(-\frac{(x - V^* t/R_d)^2}{4D_x t/R_d} - \frac{y^2}{4D_y t/R_d} - \frac{z^2}{4D_z t/R_d} - \lambda t\right) \tag{8}$$

The solution for a continuous point source is obtained by integrating Equation 8 with respect to time and letting $C_o Q = dM_o/dt$, or

$$C_c = \frac{1}{8\theta\sqrt{\pi^3 D_x D_y D_z}} \int_o^t C_o Q\, t^{-3/2} \exp\left(-\frac{(x - V^* t/R_d)^2}{4D_x t/R_d} - \frac{y^2}{4D_y t/R_d} - \frac{z^2}{4D_z t/R_c} - \lambda t\right) dt \tag{9}$$

Equation 9 can be rearranged slightly to

$$C_c = \frac{\exp\ \frac{1}{2}\frac{V^* x}{D_x}}{8\theta\sqrt{\pi^3 D_x D_y D_z}} \int_0^t C_o Q\ t^{-3/2} \exp\left(-\frac{R_d R^2}{4D_x t} - \frac{U^2 t}{4D_x R_d}\right) dt \qquad (10)$$

where

$$R^2 = x^2 + \frac{D_x}{D_y} y^2 + \frac{D_x}{D_z} z^2 \qquad (11)$$

and

$$U = V^* \left(1 + \frac{4D_x R_d \lambda}{V^{*2}}\right)^{1/2} \qquad (12)$$

Turner (1972) gives the solution to Equation 10 as

$$C_c = \frac{C_o Q \exp\left(\frac{1}{2}\frac{V^* x}{D_x}\right)}{8\pi\theta R\sqrt{D_y D_z}} \left\{ 2\cosh\left(\frac{1}{2}\frac{RU}{D_x}\right) + \exp\left(\frac{1}{2}\frac{RU}{D_x}\right) \operatorname{erf}\left[\frac{1}{2}\left(\frac{U^2 t}{D_x R_d}\right)^{1/2} - \frac{1}{2}\left(\frac{R_d R^2}{D_x t}\right)^{1/2}\right] - \exp\left(-\frac{1}{2}\frac{RU}{D_x}\right) \operatorname{erf}\left[\frac{1}{2}\left(\frac{U^2 t}{D_x R_d}\right)^{1/2} + \frac{1}{2}\left(\frac{R_d R^2}{D_x t}\right)^{1/2}\right] \right\} \qquad (13)$$

which can be simplified somewhat to yield

$$C_c = \frac{C_o\, Q \exp\left(\frac{1}{2}\frac{V^* x}{D_x}\right)}{8\pi\theta R\sqrt{D_y D_z}} \left\{ \exp\left(\frac{1}{2}\frac{RU}{D_x}\right) \operatorname{erfc}\left(\frac{1}{2}\frac{R_d R + Ut}{\sqrt{R_d D_x t}}\right) \right.$$

$$\left. + \exp\left(-\frac{1}{2}\frac{RU}{D_x}\right) \operatorname{erfc}\left(\frac{1}{2}\frac{R_d R - Ut}{\sqrt{R_d D_x t}}\right) \right\} \qquad (14a)$$

for $x > 0$.

For $x < 0$, only the first term, $\exp(V^* x/D_x)$, has the sign of x altered, because the value of the integral in Equation 10 is the same for both positive and negative values of x. Therefore

$$C_c = \frac{C_o\, Q \exp\left(-\frac{1}{2}\frac{V^* x}{D_x}\right)}{8\pi\theta R\sqrt{D_y D_z}} \left\{ \exp\left(-\frac{1}{2}\frac{RU}{D_x}\right) \operatorname{erfc}\left(\frac{1}{2}\frac{R_d R + Ut}{\sqrt{R_d D_x t}}\right) \right.$$

$$\left. + \exp\left(-\frac{1}{2}\frac{RU}{D_x}\right) \operatorname{erfc}\left(\frac{1}{2}\frac{R_d R - Ut}{\sqrt{R_d D_x t}}\right) \right\} \qquad (14b)$$

for $x < 0$.

The steady-state solution for a continuous point source is found from Equation 8 as (Hunt, 1978)

$$C_{c,\infty} = \int_0^\infty C_i \, dt \tag{15}$$

and the limit of Equation 12 as $t \to \infty$ is

$$C_{c,\infty} = \frac{C_o Q}{4\pi\theta R \sqrt{D_y D_z}} \exp\left(\frac{1}{2}\,\frac{V^* x}{D_x} - \frac{UR}{D_x}\right) \tag{16}$$

Equations 14 and 16 describe the transient and steady-state concentration distributions arising from a continuous point source in an infinite aquifer with uniform ground-water flow.

Assumptions and Limitations

Equations 14 and 16 can be used to calculate the concentrations in a leachate plume under the following assumptions and limitations:

1. The ground-water flow regime is completely saturated.
2. All aquifer properties are constant and uniform throughout the aquifer.
3. The ground-water flow is horizontal, continuous, and uniform throughout the aquifer.
4. The aquifer is infinite in extent.
5. The leachate source is a point located at the origin of the coordinate system.
6. The mass flow rate of the source is constant.
7. At zero time the concentration of leachate in the aquifer is zero.

The assumptions of an infinite aquifer depth and a uniform source mass rate can be overcome by using the principles of superposition in space and

time, respectively (Walton, 1962). Both of these provisions have been incorporated in the computer program described in the next section.

Superposition

The differential equation describing component mass concentration in a porous medium, Equation 1, is a linear partial differential equation. The principal of superposition can be used directly to solve complex ground-water contamination problems in terms of the simplier solutions described above. Unfortunately, the scattered applications of this principle are not explained in any single reference. Some texts indicate that superposition means that any sum of solutions is also a solution. Superposition is commonly used to generate a linear no-flow boundary condition through the use of "image wells" or to simulate multiple sources and sinks (Walton; 1962, 1970). The principle of superposition is also complicated by referring to the "Duhamel theorem," the "Faltung integral," and/or "convolution integrals." These terms often have no apparent physical interpretation. For the purposes of this report, "superposition in space" will refer to the approximation of sources of finite area or volume as the sum of a finite number of point sources or the generation of no-flow boundaries using image wells. "Superposition in time" will refer to the approximation of a variable source rate of contamination as the sum of a finite number of constant source rates distributed in time.

The three-dimensional solutions presented above can be used to simulate aquifers of finite width or depth or sources of finite volume. Applications of this type require a thorough understanding of the physical interpretation of the principal of superposition.

However, some applications are relatively straight forward, and the computer program provides for the approximation of a non-uniform source rate

using superposition in time. Multiple sources and aquifers of finite thickness are also included using superposition in space.

Consider the variable source of contamination shown in Figure 1. The solutions of the governing differential equation presented in this report are of the form

$$C(x,z,t) = C_o Q' \, f(x,z,t) = \dot{Q}' \, f(x,z,t) \tag{17}$$

where Q' is the source mass rate per unit length. The principle of superposition in time can be written for any position as

$$C(x,z,t) = \sum_{i=1}^{n} \dot{Q}'_i \, f(x,z,t_i) \tag{18}$$

Now, the variable rate schedule shown in Figure 1a can be decomposed into a series of positive and negative mass rates as shown in Figure 1b. The concentration at a point x,y,z at the end of the simulation, t_s, can be evaluated as

$$\begin{aligned} C(x,y,z,t) = {} & \dot{Q}'_1 \, f(x,y,z,t_1) - \dot{Q}'_1 \, f(x,y,z,t_2) \\ & + \dot{Q}'_2 \, f(x,y,z,t_2) - \dot{Q}'_2 \, f(x,y,z,t_3) \\ & + \dot{Q}'_3 \, f(x,y,z,t_3) - \dot{Q}'_3 \, f(x,y,z,t_4) \\ & + \dot{Q}'_4 \, f(x,y,z,t_4) \end{aligned} \tag{19}$$

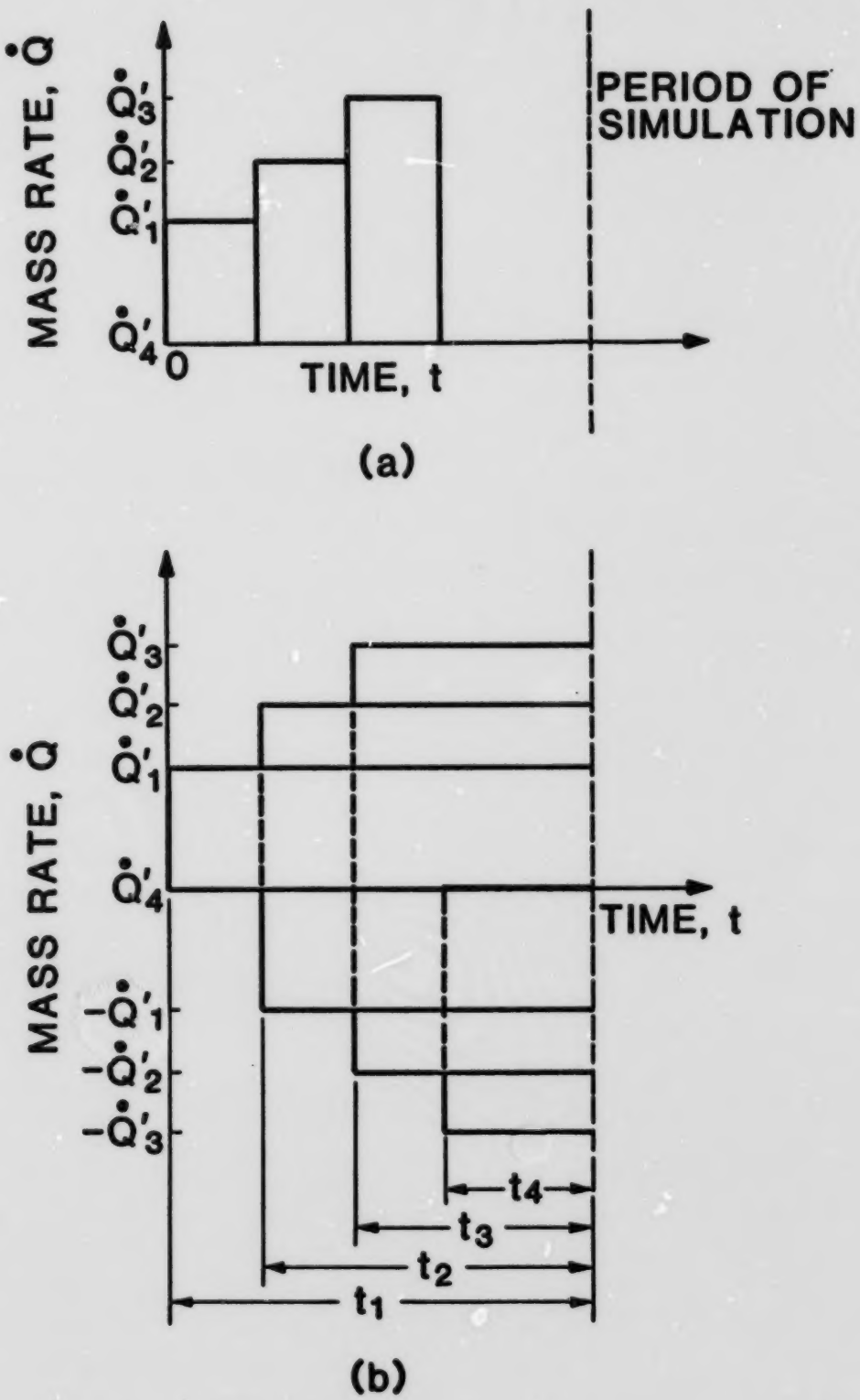


Figure 1. Decomposition of a variable source rate using superposition in time.

In general terms

$$C(x,y,z,t_s) = \sum_{i=1}^{n} (\dot{Q}'_i - \dot{Q}'_{i-1}) \; f(x,y,z,t_i) \qquad (20)$$

with $\dot{Q}'_0 = 0$

Note the time corresponding to a given source rate, t_i, is the period beginning with the start of the given rate to the end of the simulation period; time is not the duration of a given rate. For ease of application, Equation 20 can be rewritten as

$$C(x,y,z,t_s) = \sum_{k=1}^{n} (\dot{Q}'_k - \dot{Q}'_{k-1}) \; f(x,y,z,t_s - t_{k-1}) \qquad (21)$$

where t_{k-1} is the time corresponding to the end of mass rate Q_{k-1} or the beginning of rate Q_k with $Q_0 = 0$ and $t_0 = 0$.

A continuous non-uniform rate schedule may be approximated as closely as desired by increasing the number of discrete rates in the source rate schedule. In theory an infinite number of discrete rates would be required. An understanding of the physical problem and the assumptions incorporated in the mathematical model are the best guidelines for decomposing a continuous non-uniform source of contamination.

The influence of geohydrologic boundaries on the movement of a tracer is similar to the influence of these boundaries on the drawdown response of an aquifer to pumping. The applications of image well theory described by Walton (1962, 1970) can be extended to the horizontally-averaged solution to the solute transport problem considered in this report. The following discussion

parallels Walton's examples of the use of image wells to account for barrier boundaries.

Consider the contaminant plume which would exist if the aquifer were of infinite depth as shown in Figure 2a. If the contaminant plume was to intersect an impermeable base of the aquifer as shown in Figure 2b, the vertical concentration gradient must change since there can be no transport of mass across the boundary as a result of dispersion. In mathematical terms

$$D_z \frac{\partial C}{\partial z} = 0$$

at $z = B$. Now, if an imaginary, or image, source were placed across the boundary at a distance equal to the depth of the aquifer, as shown in Figure 2c, this source would create a concentration gradient from the boundary to the image water table equal to the concentration gradient from the boundary to the real water table. A "concentration divide" would be established at boundary, and the no-transport boundary condition ($\partial C/\partial z = 0$) would be satisfied.

The imaginary system of a contaminant source and its image in an aquifer of infinite depth satisfies the boundary conditions dictated by the finite depth system. The resultant concentration distribution is the sum of concentrations in both the real and image systems as shown in Figure 2d.

In theory an infinite number of image systems may be required. For example, if the plume in the infinite system intersects the water table in the image system a second no-transport boundary is encountered as shown in Figure 3. This boundary can be handled by introducing another image system across the imaginary boundary and equidistant from the first image system. This process of adding image systems could be repeated indefinitely. In practice

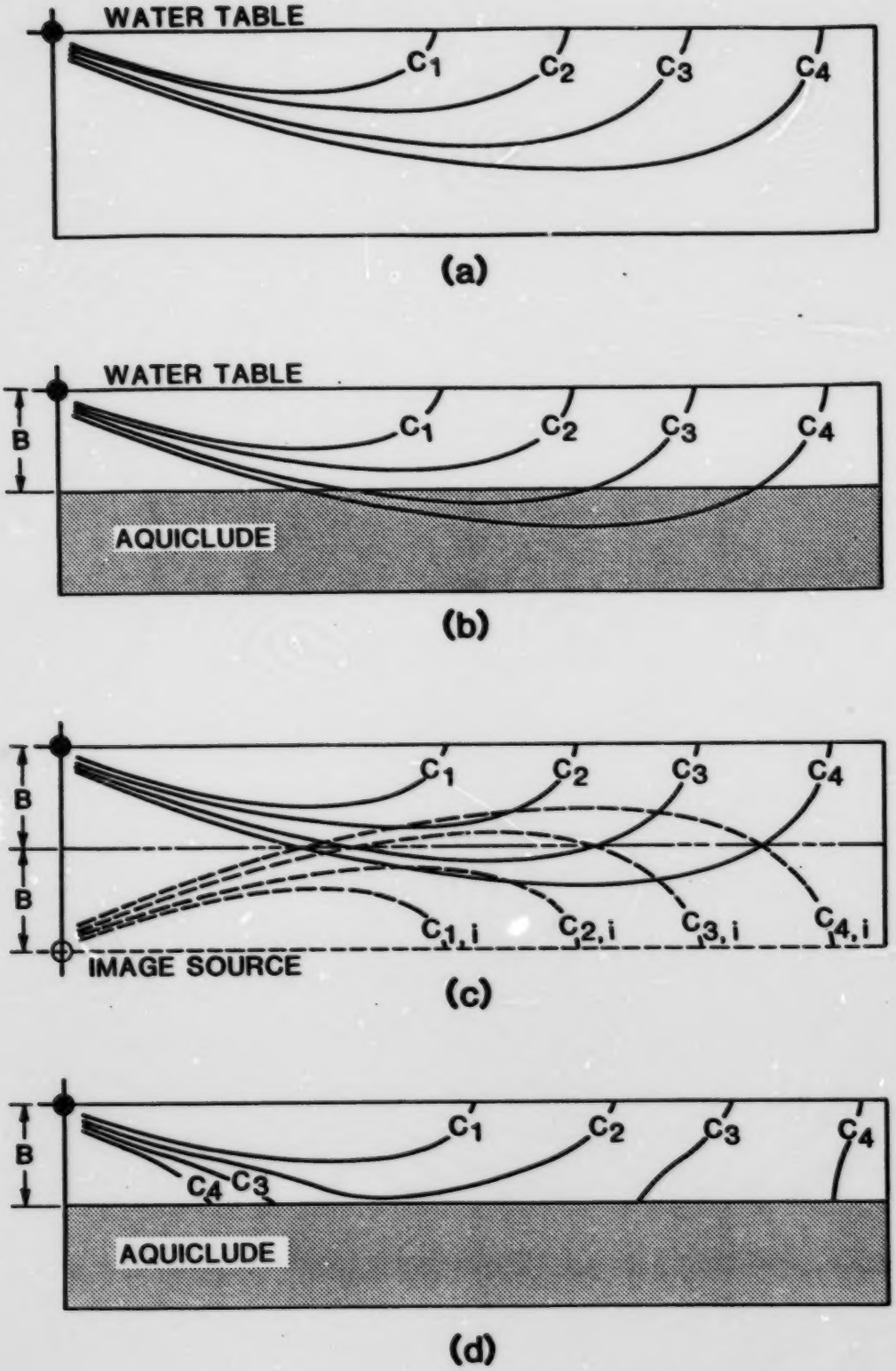
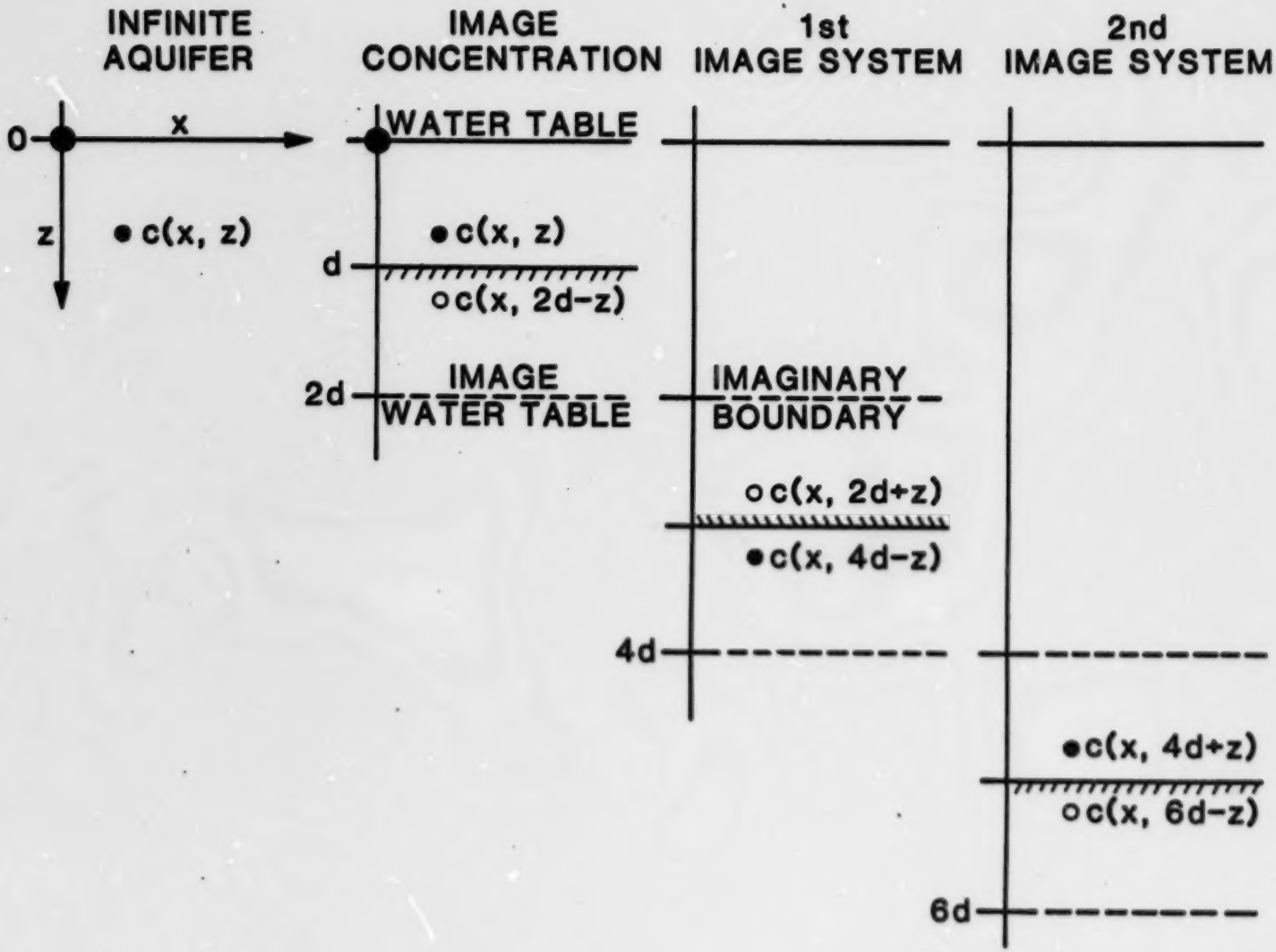


Figure 2. Use of image sources to account for aquifers of finite depth.



$$c'(x, z) = c(x, z) + \sum_{n=1}^{\infty} [c(x, 2nd - z) + c(x, 2nd + z)]$$

Figure 3. Superposition in space to account for barrier boundaries.

only a few image systems are required. The computer program automatically introduces an appropriate number of image systems.

SECTION II

COMPUTER PROGRAM

The computer program evaluates the analytical solution of the differential equation describing concentration distributions in a three-dimensional plume with uniform ground-water flow. The program has been designed for interactive use and requires input data under two modes of operation -- "Basic Input Data" and "Edit."

Basic Input Data

Basic input data are required to initiate a new problem using the PLUME3D program. The user is prompted for the required data through a series of input commands described below. Numeric data may be entered through the keyboard with or without decimal points and multiple data entries should be separated by comma(s). The first basic input command is:

```
ENTER TITLE
?
```

Any valid keyboard characters can be used. The first 60 characters will be retained for further problem identification.

The next three input commands define the units for all variables used in the calculations. Any consistent set of units may be used.

```
ENTER UNITS FOR LENGTH (2 CHARACTERS)
?
```

Any valid keyboard characters can be used. The first two characters will be retained for identifying the units of the length dimension which may be required for other input data or output listings.

```
ENTER UNITS FOR TIME (2 CHARACTERS)
?
```

Any valid keyboard characters can be used. The first two characters will be retained for identifying the units of the time dimension which may be required for other input data or output listings.

```
ENTER UNITS FOR CONCENTRATION (6 CHARACTERS)
?
```

The first six characters of any valid keyboard entries will be retained for identifying the concentration units for data input and output.

The remaining input commands are used to initialize all variables for a given problem. They include both aquifer and contaminant parameters. Input data errors which may interrupt the computational sequence are detected by the program, and a command is issued to reenter the data for the appropriate variable.

```
ENTER SATURATED THICKNESS, (0 FOR INFINITE THICKNESS), L
?
```

The saturated thickness must be entered in the units requested with dimensions of L. If a zero or negative value is entered, the calculations will be carried out assuming an aquifer of infinite depth. The program automatically includes up to 20 image wells for aquifers of finite depth.

```
ENTER AQUIFER POROSITY
?
```

Enter the volume void fraction.

```
ENTER SEEPAGE VELOCITY, L/t
?
```

The seepage, or interstitial, velocity must be entered with dimensions of L/t in the units requested. Numerical values must be greater than zero.

```
ENTER RETARDATION COEFFICIENT
?
```

The retardation coefficient includes the effects of absorption of the tracer on the solid matrix. The numerical value must be greater than 1.0, or equal to 1.0 if absorption is neglected.

ENTER X DISPERSION COEFFICIENT, SQ L/t
?

Dispersion coefficients have dimensions of L^2/t and must be entered in the units requested. Numerical values must be greater than zero. The next two commands will ask for the Y and Z dispersion coefficients, respectively. They also have dimensions of L^2/T and must be entered in the units requested. Numerical values must be greater than zero.

ENTER Y DISPERSION COEFFICIENT, SQ L/t
?

ENTER Z DISPERSION COEFFICIENT, SQ L/t
?

The subsequent command is:

ENTER DECAY CONSTANT, 1/t
?

The first order decay constant has dimensions of 1/t and must be entered in the units requested. The decay constant must be greater than, or equal to, zero.

SELECT TRANSIENT OR STEADY-STATE SOLUTION
TR FOR TRANSIENT SOLUTION
SS FOR STEADY-STATE SOLUTION
?

Selection of the transient solution also allows the approximation of a nonuniform rate schedule by a series of uniform rates. Approximation is accomplished through superposition of a series of uniform rates. If steady-state solution is chosen, the steady state concentration will be evaluated.

ENTER THE NUMBER OF SOURCES (MAXIMUM OF N)
?

The number of sources of contaminant should be entered. The value entered must be greater than zero.

MASS RATES HAVE UNITS OF (M/L^3) (L^3/t)
TIME HAS UNITS OF t

This statement reminds the user of the units that will be used for mass rates and for time. All mass rates and time values entered must be in these units.

The next three commands will be repeated for each source.

ENTER X, Y, AND Z COORDINATES OF SOURCE I (L)
?

The input units for the coordinates must be in the units requested. The Z-coordinate must be greater than or equal to zero.

If the transient solution was chosen the following two commands will be issued.

ENTER THE NUMBER OF RATES FOR SOURCE I (MAXIMUM OF N)
?

The number of uniform rates used to approximate a nonuniform rate schedule for this source is entered. The value must be greater than zero.

SOURCE I, RATE J STARTS AT TIME t
ENTER MASS RATE AND ENDING TIME
?

The source mass rate is entered in units of concentration times the volumetric rate. Note the actual source concentration and rate are not required, but the units must be consistent. The time units must also be consistent.

If the steady-state solution has been selected, the following command will be entered instead of the two previously listed commands.

ENTER STEADY-STATE MASS RATE I
?

The next three basic input commands are used to define the matrix of observation points, or coordinates at which concentration will be evaluated.

ENTER XFIRST, XLAST, DELTAX (L)
?,?,?

The input units for the coordinates must also be in the units requested. A zero entry for DELTAX will result in a single X-coordinate observation.

Results of calculations for multiple X-coordinates will be listed from XFIRST to XLAST.

ENTER YFIRST, YLAST, DELTAY (L)
?,?,?

Any of the numerical values used to define the Y-coordinates of observation points may be positive or negative.

ENTER ZFIRST, ZLAST, DELTAZ (L)
?,?,?

Both ZLAST and ZFIRST must be greater than or equal to zero and less than or equal to the saturated thickness.

ENTER PLANE FOR SECTIONING AQUIFER (XY, XZ, OR YZ)
?

The selection of a particular plane determines the presentation of the output of the program. Concentrations at the specified coordinates in the selected plane will be printed for a constant value of the third coordinate.

ENTER TFIRST, TLAST, DELTAT (t)
?,?,?

The beginning value and ending value of the time interval of contaminant transport being modeled is entered. Both TFIRST and TLAST must be positive values in the units requested. A zero entry for DELTAT will result in model output at a single value of time.

Edit Commands

Once the basic input data have been entered, the problem as currently defined is listed and the program enters the "edit" mode. The edit commands are listed in Table 1 and are also listed the first time the program enters the edit mode. The request for information is:

ENTER NEXT COMMAND ?

One of the reponses from Table 1 should be given. If the response is incorrect or improperly formulated the statement

ERROR IN LAST COMMAND -- REENTER ?

is issued. Error messages for invalid numerical data will be issued as described under the Basic Input Commands. The request for information will be repeated until one of the responses MU, LI, RN, NP, or DN is entered.

MU will list the table of edit commands.

LI will list the problem as currently defined.

RN will initiate the calculation of concentrations and print the results.

NP will request a complete new problem using the "Basic Input Data" dialog.

DN will terminate the program.

Although many tests for valid input data and properly formulated edit commands have been embedded in the program, the user is encouraged to correct "keyboard errors" before the data are transmitted. These precautions will serve to minimize the frustration of program termination as a result of fatal errors during execution of the numerical computations.

Table 1

EDIT COMMANDS

Command	Variable changed/Execution
ST	Saturated Thickness
PO	Porosity
VX	Seepage Velocity
RD	Retardation Coefficient
DE	Decay Constant
DX	X-Dispersion Coefficient
DY	Y-Dispersion Coefficient
DZ	Z-Dispersion Coefficient
RT	Source Rate Schedule
OB	Observation Points
XC	X-Coordinates
ZC	Z-Coordinates
YC	Y-Coordinates
TC	Observation Times
AS	Aquifer Sectioning
CS	Change Solution/Sources
MU	Menu of Edit Commands
LI	List Input Data
RN	Run
NP	New Problem
DN	Done

SECTION III

APPLICATIONS

The example problems presented in this document are based on the dispersal of chromium discharged to the ground water in southeastern Nassau County, New York. The hydrogeology and history of contamination have been documented by Perlmutter and Lieber (1970) and will be briefly summarized in the following paragraphs.

Site Location

The general area of the documented case history of the concentrations of contaminants in ground-water is in southeastern Nassau County, Long Island, New York (Figure 4). The detailed study area included an industrial park at South Farmingdale. During World War II, the industrial park was occupied by an aircraft company whose cadmium and chromium enriched metal plating waste was the source of much of the heavy-metal contamination in a shallow glacial aquifer.

Hydrogeology

The upper glacial aquifer which is addressed in this document extends from the water table, at depths ranging from 0 to 15 feet below the ground surface, to the top of the deeper Magothy aquifer, at depths of 80 to 140 feet below the ground surface. The upper unit consists of beds and lenses of fine to coarse sand and gravel. In some parts of the aquifer, thin lenses of fine to medium sand, as well as some silt, are interbedded with the coarse

material. Data from scattered well borings indicate that the lower 8 to 10 feet of the upper unit may consist of silty and sandy clays.

The principal direction of ground-water flow in the upper glacial unit is from north to south. The regional hydraulic gradient is approxmately 0.0025 ft/ft and the average coefficient of permeability is assumed to be 1,600 gal/day/ft^2 with an average total porosity of 0.35 (Perlmutter and Lieber, 1970). The ground-water movement is horizontal throughout the area of interest, with the exception of local recharge and discharge areas where vertical or oblique flow may be predominant.

Plating-Waste Contamination

The chromium contamination in the upper glacial unit was derived principally from the disposal of metal-plating and anodizing waste water to unlined basins. During World War II, and for several years thereafter, essentially untreated plating-water effluent was recharged to the aquifer through these disposal basins. Perlmutter and Lieber (1970) estimated that during the early 1940's as much as 200,000 to 300,000 gallons of effluent containing approximately 52 pounds of chromium was discharged daily into the upper glacial unit. At the end of World War II, the amount of plating-waste effluent was decreased substantially and a chromium removal unit was installed in 1949.

By 1949, or approximately 9 years after the start of plating-waste disposal, a plume of contaminated water had migrated southward approximately 3,900 feet with a maximum width of about 850 feet. Maximum observed chromium concentration was approximately 40 mg/l.

Since 1949 the chromium concentrations decreased substantially in nearly all sections of the sections of the plume. The maximum observed chromium

concentration was about 10 mg/l in 1962. However, the plume of contaminated water had migrated further southward beyond Massapequa Creek, a small stream which serves as a natural drain for part of the contaminated ground water. The plume was approximately 4,300 feet long with a maximum width of almost 1000 feet in 1962.

Analytical Model

The data required for the analytical two-dimensional plume model are listed in Table 2. The following paragraphs will discuss the estimation of these model parameters based on the limited field data, judgement, and experience.

All input data for the computer program must be in consistent units. For the example problems, the following system of units will be selected: length in feet, time in days, and concentration in mg/l.

Assuming that the water table is located approximately at the ground surface, the average saturated thickness of the upper glacial unit can be estimated from the depth to the top of the deeper Magothy aquifer. Thus, the average saturated thickness, S_t, will be taken as:

$$S_t = \frac{80 + 140}{2} = 110 \text{ ft}$$

The superficial, or Darcy, velocity can be estimated from the average coefficient of permeability, K, and the regional hydraulic gradient, dh/dx. From Darcy's law

$$V = - K \frac{dh}{dx}$$

TABLE 2

Input Data Required for the Analytical
Three-Dimensional Plume Model

Title - Units for length, time, and concentration

Saturated thickness (for aquifer of finite depth)

Effective porosity

Ground water interstitial velocity

Retardation coefficient

Longitudinal dispersion coefficient

Transverse dispersion coefficient

Vertical dispersion coefficient

First-order decay constant

Type of solution (transient or steady-state)

Number of sources

Location and rate schedules for each source

Coordinates of observation points

Observation times (for transient solution)

where the minus sign indicates that the flow is in the direction of decreasing hydraulic head. Thus

$$V = \frac{1600 \text{ gal}}{\text{day ft}^2} \frac{\text{ft}^3}{7.48 \text{ gal}} \frac{0.0025 \text{ ft}}{\text{ft}}$$

and

$$V = 0.52 \text{ ft/day}$$

The average interstitial, or pore, velocity, V*, can be estimated by assuming that the areal porosity is equal to the volume porosity, θ, and

$$V^* = \frac{V}{\theta}$$

Using the estimated value of effective porosity

$$V^* = \frac{0.52 \text{ ft/day}}{0.35}$$

or

$$V^* = 1.5 \text{ ft/day}$$

The longitudinal, transverse and vertical dispersion coefficients are the most difficult parameters to estimate. Typical values for dispersivities, as well as hydraulic conductivity, are summarized in Table 3. Based on the results of a numerical model for the same site, Pinder (1973) estimated longitudinal and transverse dispersivities as α_x = 69.9 feet and α_y = 14.0 feet, respectively. Considering the interbedding of finer materials in the

TABLE 3

Typical Values of Aquifer Properties

(after Yeh, 1981)

Parameter	Material		
	Clay	Silt	Sand
Bulk density, lb/ft^3	87.36 - 137.2	80.50 - 112.3	73.63 - 98.59
Effective porosity	0.03 - 0.05	0.05 - 0.10	0.10 - 0.30
Hydraulic Conductivity, $gal/day/ft^2$	0.01 - 0.1	1 - 10	100 - 100,000
Dispersivity, ft			
Longitudinal	0.1 - 1.0	1 - 10	10 - 100
Transverse	0.01 - 0.1	0.1 - 1.0	1.0 - 10
Vertical	0.01 - 0.1	0.1 - 1.0	1 - 10

shallow upper unit together with the typical values of longitudinal and vertical dispersivities in Table 3, the vertical dispersivity will be assumed to be approximately two orders of magnitude smaller than the longitudinal dispersivity. Thus α_z = 0.7 ft, and the dispersion coefficients are evaluated as

$$D_x = \alpha_L V^* = (69.9 \text{ ft})(1.5 \text{ ft/day}) = 105 \text{ ft}^2/\text{day}$$

$$D_y = \alpha_y V^* = (14.0 \text{ ft})(1.5 \text{ ft/day}) = 21.0 \text{ ft}^2/\text{day}$$

and

$$D_z = \alpha_z V^* = (0.7 \text{ ft})(1.5 \text{ ft/day}) = 1.05 \text{ ft}^2/\text{day}$$

Chromium is believed to be a conservative material in this system (Perlmutter and Lieber, 1970), and both adsorption and chemical/biological decay can be neglected. Therefore, $K_d = 0$ (or $R_d = 1$) and $\lambda = 0$.

The last set of model input data to be specified is the source mass/rate schedule. A steady mass rate of 52 lb/day of chromium will be assumed from the time of initial injection to mid-1949, or approximately 2800/days. At this time, the actual extent of the contamination was estimated; and the chromium removal unit was installed. The actual volumetric rate and concentration of the source of chromium contamination are not required. However, the units of the source term must be consistent with other parameters in the model.

The mass rate of chromium can be converted to units of concentration times volume rate per unit width of aquifer as follows:

TABLE 4

Model Input Data for Example Problem 1.

Title:	Hexavalent Chromium Plume--Example 1
Units for length:	ft
Units for time:	dy
Units for concentration:	mg/l
Saturated thickness:	110 ft
Effective porosity:	0.35
Interstitial velocity:	1.5 ft/dy
Retardation coefficient:	1.0
x-dispersion coefficient	105 ft^2/dy
y-dispersion coefficient	21.0 ft^2/dy
z-dispersion coefficient	1.05 ft^2/dy
First-order decay constant:	0/dy
Source/rate schedule	
Number of sources:	1
Location of source:	x = 0, y = 0, z = 0
Number of rates:	1
Mass rate and time:	833,586 (mg/l)(ft^3/dy) from 0 to 2800 dy

$$\frac{52 \text{ lb}}{\text{day}} \quad \frac{454 \times 10^3 \text{ mg}}{\text{lb}} \quad \frac{\text{ft}^3}{28.321 \text{ liter}} = 833{,}586 \text{ (mg/liter)(ft}^3\text{/day)}$$

for 2800 days.

The model input data for this example problem are summarized in Table 4. The input data dialog for the example problem and the model results are included in Appendix A.

This example problem is not intended to serve as a verification of a model. The example illustrates the type of input data required, and some methods for estimating input data. The input data dialog and model results can be used to partially test the model code. The analytical model and the computer program are tools which can aid in the analysis of ground-water contamination problems. The user must select the best tools for the problem at hand based on a sound understanding of the principles of ground-water hydrology, the physical problems, and the limitations of the mathematical model(s).

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APPENDIX A

Input Data Dialog and Results for Example Problem

```
ENTER TITLE
?HEXAVALENT CHROMIUM PLUME -- EXAMPLE 1


ENTER UNITS FOR LENGTH (2 CHARACTERS)
?FT


ENTER UNITS FOR TIME (2 CHARACTERS)
?DY


ENTER UNITS FOR CONCENTRATION (6 CHARACTERS)
?MG/L


ENTER SATURATED THICKNESS (Ø FOR INFINITE THICKNESS), FT
?11Ø.


ENTER AQUIFER POROSITY
?Ø.35


ENTER SEEPAGE VELOCITY, FT/DY
?1.5


ENTER RETARDATION COEFFICIENT
?1.Ø


ENTER X DISPERSION COEFFICIENT, SQ FT/DY
?1Ø5.


ENTER Y DISPERSION COEFFICIENT, SQ FT/DY
?21.


ENTER Z DISPERSION COEFFICIENT, SQ FT/DY
?1.Ø5


ENTER DECAY CONSTANT, 1/DY
?Ø.


SELECT TRANSIENT OR STEADY-STATE SOLUTION
   TR FOR TRANSIENT SOLUTION
   SS FOR STEADY-STATE SOLUTION
?TR
```

```
ENTER THE NUMBER OF SOURCES (MAXIMUM OF 10 )
?1


MASS RATES HAVE UNITS OF (MG/L  ) (CU FT/DY)
TIME HAS UNITS OF DY

ENTER X, Y, AND Z COORDINATES OF SOURCE 1 (FT)
?,?,?0.,0.,0.


ENTER THE NUMBER RATES FOR SOURCE 1 (MAXIMUM 0F 10)
?1


SOURCE  1, RATE  1 STARTS AT     0.0 DY
ENTER MASS RATE AND ENDING TIME
?,?833586.,2800.


ENTER XIRST, XLAST, DELTAX  (FT)
?,?,?600.,3600.,600.


ENTER YFIRST, YLAST, DELTAY  (FT)
?,?,?450.,-450.,150.


ENTER ZFIRST, ZLAST, DELTAZ  (FT)
?,?,?0.,110.,55.


ENTER PLANE FOR SECTIONING AQUIFER (XY,XZ,OR YZ)
?XY


ENTER TFIRST, TLAST, DELTAT  (DY)
?,?,?2800.,2800.,0.
```

PLUME3D
VERSION 2.01
PAGE 1

HEXAVALENT CHROMIUM PLUME -- EXAMPLE 1

SATURATED THICKNESS, (FT)	110.0000
SEEPAGE VELOCITY, (FT/DY)	1.5000
X DISPERSION COEFFICIENT (FT**2/DY)	105.0000
Y DISPERSION COEFFICIENT (FT**2/DY)	21.0000
Z DISPERSION COEFFICIENT (FT**2/DY)	1.0500
POROSITY	.3500
RETARDATION COEFFICIENT	1.0000
FIRST ORDER DECAY CONSTANT (1/DY)	0.0000

SOURCE/RATE SCHEDULE (MG/L)(CU FT/DY)

	SOURCE			RATE	MASS	TIME (DY)	
NO	X (FT)	Y (FT)	Z (FT)	NO	RATE	START	END
1	0.00	0.00	0.00	1	833586.00	0.00	2800.00

OBSERVATION POINTS (FT), AND TIMES (DY)

XFIRST =	600.00	XLAST =	3600.00	DELX =	600.0000
YFIRST =	450.00	YLAST =	-450.00	DELY =	150.0000
ZFIRST =	0.00	ZLAST =	110.00	DELZ =	55.0000
TFIRST =	2800.00	TLAST =	2800.00	DELT =	0.0000

AQUIFER SECTIONED IN XY PLANE

```
MENU OF EDIT COMMANDS

SATURATED THICKNESS              ST      OBSERVATION POINTS    OB
POROSITY                         PO      X COORDINATES         XC
SEEPAGE VELOCITY                 VX      Y COORDINATES         YC
RETARDATION COEFFICIENT          RD      Z COORDINATES         ZC
X DISPERSION COEFFICIENT         DX      OBSERVATION TIMES     TC
Y DISPERSION COEFFICIENT         DY      AQUIFER SECTIONING    AS
Z DISPERSION COEFFICIENT         DZ      NEW PROBLEM           NP
DECAY CONSTANT                   DE      MENU OF COMMANDS      MU
SOURCE/RATE SCHEDULE             RT      LIST INPUT DATA       LI
CHANGE SOLUTION/SOURCES          CS      RUN CALCULATIONS      RN
                                         DONE                  DN


ENTER NEXT COMMAND
?RN
```

HEXAVALENT CHROMIUM PLUME -- EXAMPLE 1

CONCENTRATION DISTRIBUTION AT 2800.00 DY (MG/L)

Z = 0.00 FT

Y(FT) \ X(FT)	600.00	1200.00	1800.00	2400.00	3000.00	3600.00
450.00	1.1622	3.7737	6.0164	7.2392	7.3914	6.2020
300.00	10.5229	16.8523	17.7165	16.6967	14.7097	11.3227
150.00	62.9100	46.6486	35.3420	28.0852	22.4262	16.3152
0.00	134.5398	67.2738	44.8561	33.5146	25.8517	18.4413
-150.00	62.9100	46.6486	35.3420	28.0852	22.4262	16.3152
-300.00	10.5229	16.8523	17.7165	16.6967	14.7097	11.3227
-450.00	1.1622	3.7737	6.0164	7.2392	7.3914	6.2020

HEXAVALENT CHROMIUM PLUME -- EXAMPLE 1

CONCENTRATION DISTRIBUTION AT 2800.00 DY (MG/L)

Z = 55.00 FT

Y(FT) \ X(FT)	600.00	1200.00	1800.00	2400.00	3000.00	3600.00
450.00	.4395	1.8379	3.5409	4.8545	5.4148	4.8044
300.00	2.9774	7.3832	9.9691	10.9487	10.6514	8.7202
150.00	12.3888	18.7138	19.2646	18.1440	16.1159	12.5181
0.00	21.5268	26.0413	24.1684	21.5383	18.5286	14.1310
-150.00	12.3888	18.7138	19.2646	18.1440	16.1159	12.5181
-300.00	2.9774	7.3832	9.9691	10.9487	10.6514	8.7202
-450.00	.4395	1.8379	3.5409	4.8545	5.4148	4.8044

HEXAVALENT CHROMIUM PLUME -- EXAMPLE 1

CONCENTRATION DISTRIBUTION AT 2800.00 DY (MG/L)

Z = 110.00 FT

Y(FT) \ X(FT)	600.00	1200.00	1800.00	2400.00	3000.00	3600.00
450.00	.0755	.5147	1.4813	2.6624	3.5125	3.4326
300.00	.3219	1.6706	3.7892	5.7406	6.7651	6.1702
150.00	.8561	3.5798	6.8411	9.2287	10.0941	8.8018
0.00	1.2145	4.6664	8.3725	10.8380	11.5486	9.9142
-150.00	.8561	3.5798	6.8411	9.2287	10.0941	8.8018
-300.00	.3219	1.6706	3.7892	5.7406	6.7651	6.1702
-450.00	.0755	.5147	1.4813	2.6624	3.5125	3.4326

ENTER NEXT COMMAND
?YC

ENTER YFIRST, YLAST, DELTAY (FT)
?,?,?0.,0.,0.

ENTER NEXT COMMAND
?ZC

ENTER ZFIRST, ZLAST, DELTAZ (FT)
?,?,?0.,110.,20.

ENTER NEXT COMMAND
?AS

ENTER PLANE FOR SECTIONING AQUIFER (XY,XZ,OR YZ)
?XZ

ENTER NEXT COMMAND
?LI

HEXAVALENT CHROMIUM PLUME -- EXAMPLE 1

SATURATED THICKNESS, (FT)	110.0000
SEEPAGE VELOCITY, (FT/DY)	1.5000
X DISPERSION COEFFICIENT (FT**2/DY)	105.0000
Y DISPERSION COEFFICIENT (FT**2/DY)	21.0000
Z DISPERSION COEFFICIENT (FT**2/DY)	1.0500
POROSITY	.3500
RETARDATION COEFFICIENT	1.0000
FIRST ORDER DECAY CONSTANT (1/DY)	0.0000

SOURCE/RATE SCHEDULE (MG/L)(CU FT/DY)

	SOURCE			RATE	MASS	TIME (DY)	
NO	X (FT)	Y (FT)	Z (FT)	NO	RATE	START	END
1	0.00	0.00	0.00				
				1	833586.00	0.00	2800.00

OBSERVATION POINTS (FT), AND TIMES (DY)

XFIRST =	600.00	XLAST =	3600.00	DELX =	600.0000
YFIRST =	0.00	YLAST =	0.00	DELY =	0.0000
ZFIRST =	0.00	ZLAST =	110.00	DELZ =	20.0000
TFIRST =	2800.00	TLAST =	2800.00	DELT =	0.0000

AQUIFER SECTIONED IN XZ PLANE

ENTER NEXT COMMAND
?RN

HEXAVALENT CHROMIUM PLUME -- EXAMPLE 1

CONCENTRATION DISTRIBUTION AT 2800.00 DY (MG/L)

Y = 0.00 FT

Z(FT) \ X(FT)	600.00	1200.00	1800.00	2400.00	3000.00	3600.00
0.00	134.5398	67.2738	44.8561	33.5146	25.8517	18.4413
20.00	101.2264	58.9650	41.2153	31.5259	24.6661	17.7508
40.00	47.1345	40.1774	32.1269	26.3553	21.5287	15.9102
60.00	16.1375	22.0086	21.6384	19.9389	17.5144	13.5252
80.00	4.7086	10.3607	13.3152	14.3904	13.9192	11.3590
100.00	1.5140	5.2569	8.9261	11.2473	11.8248	10.0831
110.00	1.2145	4.6664	8.3725	10.8380	11.5486	9.9142

ENTER NEXT COMMAND
?DN

STOP

APPENDIX B

Listing of Source Code for PLUME3D

```
C     PLUME3D                                                          PL3D001
C     VERSION 2.02                                                     PL3D002
C     THREE-DIMENSIONAL PLUMES IN UNIFORM GROUND-WATER FLOW            PL3D003
C        JAN WAGNER                                                    PL3D004
C        SCHOOL OF CHEMICAL ENGINEERING                                PL3D005
C        OKLAHOMA STATE UNIVERSITY                                     PL3D006
C        STILLWATER, OK  74078                                         PL3D007
C        PHONE (405) 624-5280                                          PL3D008
C        MARCH, 1984                                                   PL3D009
C                                                                      PL3D010
C        REVISIONS:    2.01   23 NOV 84                               PL3D011
C                      2.02    9 DEC 84                               PL3D012
C                                                                      PL3D013
      DIMENSION COL(7),CON(7),D(3),DEL(3),XF(3),XL(3),XS(10),YS(10),   PL3D014
     1ZS(10)                                                          PL3D015
      DIMENSION IC(21),IS(4),KSEC(3),LBL(2,6),NP(3),NR(10),TITLE(30)   PL3D016
      REAL LAMBDA                                                      PL3D017
      INTEGER TITLE                                                    PL3D018
      COMMON/IO/NI,NO                                                  PL3D019
      COMMON/RATE/Q(10,12),T(10,12)                                    PL3D020
      COMMON/PHYPRO/ALPHA,BETA,GAMMA,DX,LAMBDA,PE,RD,V                 PL3D021
      DATA IC/'NP','ST','PO','VX','RD','DX','DY','DZ','DE','CS','XC', PL3D022
     1'YC','ZC','AS','TC','LI','RN','OB','RT','MU','DN'/              PL3D023
      DATA IS/'R','M','A','D'/                                         PL3D024
      DATA KAR1,KAR2,KAR3/'X','Y','Z'/                                 PL3D025
      DATA KSEC/'XY','XZ','YZ'/                                        PL3D026
      DATA LBL/'  ','(C','  ','ON','  ','TI','  ','NU','  ','ED',      PL3D027
     1'  ',') '/                                                       PL3D028
      DATA IY/'Y'/                                                     PL3D029
      DATA KSOL1,KSOL2/'TR','SS'/                                      PL3D030
C                                                                      PL3D031
C     READ DEVICE: NI       WRITE DEVICE: NO                           PL3D032
      NI=1                                                             PL3D033
      NO=1                                                             PL3D034
C                                                                      PL3D035
C     MAXIMUM NUMBER OF PRINTED COLUMNS PER PAGE IS SET TO MAXCOL      PL3D036
C        DIMENSION COL(MAXCOL),CON(MAXCOL)                             PL3D037
      MAXCOL = 7                                                       PL3D038
C                                                                      PL3D039
C     MAXIMUM NUMBER OF PRINTED ROWS PER PAGE IS SET TO MAXROW         PL3D040
      MAXROW = 40                                                      PL3D041
C                                                                      PL3D042
C     MAXIMUM NUMBER OF SOURCES IS SET TO MAXSOR                       PL3D043
C        DIMENSION XS(MAXSOR),YS(MAXSOR),ZS(MAXSOR),NR(MAXSOR)         PL3D044
      MAXSOR = 10                                                      PL3D045
C                                                                      PL3D046
C     MAXIMUM NUMBER OF SOURCE RATES FOR SUPERPOSITION IN TIME         PL3D047
C     IS SET TO MAXRT                                                  PL3D048
C        COMMON/RATE/ Q(MAXSOR,MAXRT+2),T(MAXSOR,MAXRT+2)             PL3D049
      MAXRT = 10                                                       PL3D050
C                                                                      PL3D051
C     MAXIMUM NUMBER OF IMAGE WELLS FOR SUPERPOSITION IN SPACE         PL3D052
C     IS SET TO MAXIMG                                                 PL3D053
      MAXIMG = 20                                                      PL3D054
C                                                                      PL3D055
C                                                                      PL3D056
C                                                                      PL3D057
C     INITIALIZE PROGRAM FLOW CONTROL VARIABLES                        PL3D058
   10 IEDIT = 1                                                        PL3D059
      KNTL = 1                                                         PL3D060
      NPAGE = 1                                                        PL3D061
C                                                                      PL3D062
C                                                                      PL3D063
C ***** SECTION I -- BASIC INPUT DATA                                  PL3D064
C                                                                      PL3D065
C     READ TITLE                                                       PL3D066
      WRITE(NO,15)                                                     PL3D067
   15 FORMAT(1H1,2X,'ENTER TITLE',/'   ?')                             PL3D068
      READ(NI,25) (TITLE(I), I=1,30)                                   PL3D069
   25 FORMAT(30A2)                                                     PL3D070
```

```
C                                                                       PL3D071
C     DEFINE UNITS                                                      PL3D072
      WRITE(NO,35)                                                      PL3D073
   35 FORMAT(3X,'ENTER UNITS FOR LENGTH (2 CHARACTERS)',/,'   ?')       PL3D074
      READ(NI,45) IL                                                    PL3D075
   45 FORMAT(A2)                                                        PL3D076
      WRITE(NO,55)                                                      PL3D077
   55 FORMAT(3X,'ENTER UNITS FOR TIME (2 CHARACTERS)',/,'   ?')         PL3D078
      READ(NI,45) IT                                                    PL3D079
      WRITE(NO,65)                                                      PL3D080
   65 FORMAT(3X,'ENTER UNITS FOR CONCENTRATION (6 CHARACTERS)',/,'   ?') PL3D081
      READ(NI,75) IM1,IM2,IM3                                           PL3D082
   75 FORMAT(3A2)                                                       PL3D083
C                                                                       PL3D084
C     ENTER DATA FOR FIRST PROBLEM                                      PL3D085
C                                                                       PL3D086
C     SATURATED THICKNESS                                               PL3D087
   80 IMAGE = MAXIMG/2                                                  PL3D088
   84 WRITE(NO,85) IL                                                   PL3D089
   85 FORMAT(3X,'ENTER SATURATED THICKNESS (O FOR INFINITE THICKNESS), ' PL3D090
     1A2,/,'   ?')                                                     PL3D091
      READ(NI,95,ERR=84) ST                                             PL3D092
   95 FORMAT(F10.0)                                                     PL3D093
      IF(ST.GT.0.0) GO TO 100                                           PL3D094
      IMAGE = 1                                                         PL3D095
      ST = 1.0E32                                                       PL3D096
  100 CONTINUE                                                          PL3D097
      GO TO (120,110),IEDIT                                             PL3D098
  110 IF(XF(3).LE.ST.AND.XL(3).LE.ST) GO TO 3000                        PL3D099
      WRITE(NO,115) ST,IL                                               PL3D100
  115 FORMAT(3X,'RANGE OF Z-COORDINATES IS OUTSIDE UPPER LIMIT OF',/,   PL3D101
     13X,'SATURATED THICKNESS = ',F10.4,A3)                             PL3D102
      GO TO 590                                                         PL3D103
C                                                                       PL3D104
C     POROSITY                                                          PL3D105
  120 WRITE(NO,125)                                                     PL3D106
  125 FORMAT(3X,'ENTER AQUIFER POROSITY',/,'   ?')                      PL3D107
  130 READ(NI,95,ERR=120) P                                             PL3D108
      IF(P.GT.0.0.AND.P.LT.1.0) GO TO 150                               PL3D109
      WRITE(NO,145)                                                     PL3D110
  145 FORMAT(3X,'POROSITY MUST BE GREATER THAN ZERO',                   PL3D111
     1' AND LESS THAN ONE -- REENTER',/,'   ?')                         PL3D112
      GO TO 130                                                         PL3D113
  150 GO TO (160,3000),IEDIT                                            PL3D114
C                                                                       PL3D115
C     SEEPAGE VELOCITY                                                  PL3D116
  160 WRITE(NO,165) IL,IT                                               PL3D117
  165 FORMAT(3X,'ENTER SEEPAGE VELOCITY, 'A2,'/',A2,/,'   ?')           PL3D118
  170 READ(NI,95,ERR=160) V                                             PL3D119
      IF(V.GT.0.0) GO TO 180                                            PL3D120
      WRITE(NO,175)                                                     PL3D121
  175 FORMAT(3X,'SEEPAGE VELOCITY MUST BE GREATER THAN ZERO',           PL3D122
     1' -- REENTER',/,'   ?')                                           PL3D123
      GO TO 170                                                         PL3D124
  180 GO TO (190,3000),IEDIT                                            PL3D125
C                                                                       PL3D126
C     RETARDATION COEFFICIENT                                           PL3D127
  190 WRITE(NO,195)                                                     PL3D128
  195 FORMAT(3X,'ENTER RETARDATION COEFFICIENT',/,'   ?')               PL3D129
  200 READ(NI,95,ERR=190) RD                                            PL3D130
      IF(RD.GE.1.0) GO TO 210                                           PL3D131
      WRITE(NO,205)                                                     PL3D132
  205 FORMAT(3X,'RETARDATION COEFFICIENT MUST BE GREATER THAN OR',      PL3D133
     1' EQUAL TO ONE',/,3X,' -- REENTER',/,'   ?')                      PL3D134
      GO TO 200                                                         PL3D135
  210 GO TO (220,3000),IEDIT                                            PL3D136
C                                                                       PL3D137
C     X DISPERSION COEFFICIENT                                          PL3D138
  220 WRITE(NO,225) IL,IT                                               PL3D139
  225 FORMAT(3X,'ENTER X DISPERSION COEFFICIENT, SQ ',A2,               PL3D140
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     1'/',A2,/,'   ?')                                                    PL3D141
  230 READ(NI,95,ERR=220) DX                                             PL3D142
      IF(DX.GT.0.0) GO TO 240                                            PL3D143
      WRITE(NO,235)                                                      PL3D144
  235 FORMAT(3X,'X DISPERSION COEFFICIENT MUST BE GREATER THAN ZERO',    PL3D145
     1' -- REENTER',/,'   ?')                                            PL3D146
      GO TO 230                                                          PL3D147
  240 GO TO (250,3000),IEDIT                                             PL3D148
C                                                                        PL3D149
C     Y DISPERSION COEFFICIENT                                           PL3D150
  250 WRITE(NO,255) IL,IT                                                PL3D151
  255 FORMAT(3X,'ENTER Y DISPERSION COEFFICIENT, SQ ',A2,                 PL3D152
     1'/',A2,/,'   ?')                                                   PL3D153
  260 READ(NI,95,ERR=250) DY                                             PL3D154
      IF(DY.GT.0.0) GO TO 270                                            PL3D155
      WRITE(NO,265)                                                      PL3D156
  265 FORMAT(3X,'Y  DISPERSION COEFFICIENT MUST BE GREATER THAN ZERO',   PL3D157
     1' -- REENTER',/,'   ?')                                            PL3D158
      GO TO 260                                                          PL3D159
  270 GO TO (280,3000),IEDIT                                             PL3D160
C                                                                        PL3D161
C     Z DISPERSION COEFFICIENT                                           PL3D162
  280 WRITE(NO,285) IL,IT                                                PL3D163
  285 FORMAT(3X,'ENTER Z DISPERSION COEFFICIENT, SQ ',A2,                 PL3D164
     1'/',A2,/,'   ?')                                                   PL3D165
  290 READ(NI,95,ERR=280) DZ                                             PL3D166
      IF(DZ.GT.0.0) GO TO 300                                            PL3D167
      WRITE(NO,295)                                                      PL3D168
  295 FORMAT(3X,'Z  DISPERSION COEFFICIENT MUST BE GREATER THAN ZERO',   PL3D169
     1' -- REENTER',/,'   ?')                                            PL3D170
      GO TO 290                                                          PL3D171
  300 GO TO (310,3000),IEDIT                                             PL3D172
C                                                                        PL3D173
C     FIRST-ORDER DECAY CONSTANT                                         PL3D174
  310 WRITE(NO,315) IT                                                   PL3D175
  315 FORMAT(3X,'ENTER DECAY CONSTANT, 1/',A2,/,'   ?')                  PL3D176
      READ(NI,95,ERR=310) DECAY                                          PL3D177
      GO TO (320,3000),IEDIT                                             PL3D178
C                                                                        PL3D179
C     DEFINE LOCATIONS AND RATES OF SOURCES                              PL3D180
C     INITIALIZE SOURCE/RATE ARRAYS                                      PL3D181
  320 MAXRT2 = MAXRT + 2                                                 PL3D182
      DO 330  I=1,MAXSOR                                                 PL3D183
        DO 330  J=1,MAXRT2                                               PL3D184
           Q(I,J) = 0.0                                                  PL3D185
           T(I,J) = 0.0                                                  PL3D186
  330 CONTINUE                                                           PL3D187
      JFLOW = 3                                                          PL3D188
  340 WRITE(NO,345)                                                      PL3D189
  345 FORMAT(3X,'SELECT TRANSIENT OR STEADY-STATE SOLUTION',/,           PL3D190
     16X,'TR FOR TRANSIENT SOLUTION',/,                                  PL3D191
     26X,'SS FOR STEADY-STATE SOLUTION',/,'   ?')                       PL3D192
  350 READ(NI,45) KSOL                                                   PL3D193
      IF(KSOL.EQ.KSOL1) JFLOW=1                                          PL3D194
      IF(KSOL.EQ.KSOL2) JFLOW=2                                          PL3D195
      GO TO (370,370,360), JFLOW                                         PL3D196
  360 WRITE(NO,365)                                                      PL3D197
  365 FORMAT(3X,'ERROR IN SELECTION -- REENTER'/,'   ?')                 PL3D198
      GO TO 350                                                          PL3D199
C                                                                        PL3D200
  370 WRITE(NO,375) MAXSOR                                               PL3D201
  375 FORMAT(3X,'ENTER THE NUMBER OF SOURCES (MAXIMUM OF',I3,' )',/,     PL3D202
     1'  ?')                                                            PL3D203
  380 READ(NI,385,ERR=370) FDUM                                          PL3D204
  385 FORMAT(F10.0)                                                      PL3D205
      NS=FDUM                                                            PL3D206
      IF(NS.GT.0.AND.NS.LE.MAXSOR) GO TO 400                             PL3D207
      WRITE(NO,395) MAXSOR                                               PL3D208
  395 FORMAT(3X,'NUMBER OF SOURCES MUST BE GREATER THAN ZERO '           PL3D209
     1'AND LESS THAN',I3,' -- REENTER',/,'   ?')                         PL3D210
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      GO TO 380                                                          PL3D211
  400 WRITE (NO,405) IM1,IM2,IM3,IL,IT,IT                                PL3D212
  405 FORMAT(3X,'MASS RATES HAVE UNITS OF (',3A2,') (CU ',A2,'/',A2,     PL3D213
     1 ')',/,3X,'TIME HAS UNITS OF ',A2,/)                               PL3D214
      DO 540  I=1,NS                                                     PL3D215
  414 WRITE(NO,415) I,IL                                                 PL3D216
  415 FORMAT(3X,'ENTER X, Y, AND Z COORDINATES OF SOURCE',I2,            PL3D217
     1' (',A2,')',/,'   ?,?,?')                                         PL3D218
      READ(NI,425,ERR=414) XS(I),YS(I),ZS(I)                             PL3D219
  425 FORMAT(3F10.0)                                                     PL3D220
  430 IF(ZS(I).GE.0.0.AND.ZS(I).LE.ST) GO TO 440                         PL3D221
  434 WRITE(NO,435) ST,IL                                                PL3D222
  435 FORMAT(3X,'Z-COORDINATE MUST BE GREATER THAN OR EQUAL TO ZERO',    PL3D223
     1' AND',/,3X,'LESS THAN OR EQUAL TO SATURATED THICKNESS (',         PL3D224
     2F10.4,A3,')',/,3X,' -- REENTER',/,'   ?')                          PL3D225
      READ(NI,95,ERR=434) ZS(I)                                          PL3D226
      GO TO 430                                                          PL3D227
  440 IF(JFLOW.EQ.2) GO TO 530                                           PL3D228
      Q(I,1) = 0.0                                                       PL3D229
      T(I,1) = 0.0                                                       PL3D230
  450 WRITE(NO,455) I,MAXRT                                              PL3D231
  455 FORMAT(3X,'ENTER THE NUMBER RATES FOR SOURCE',I2,                  PL3D232
     1' (MAXIMUM OF',I3,')',/,'   ?')                                    PL3D233
  460 READ(NI,465,ERR=450) FDUM                                          PL3D234
  465 FORMAT(F10.0)                                                      PL3D235
      NR(I)=FDUM                                                         PL3D236
      IF(NR(I).GT.0.AND.NR(I).LE.MAXRT) GO TO 480                        PL3D237
      WRITE(NO,475) MAXRT                                                PL3D238
  475 FORMAT(3X,'NUMBER OF RATES MUST BE GREATER THAN ZERO AND ',        PL3D239
     1'LESS THAN',I3,'  -- REENTER',/,'   ?')                            PL3D240
      GO TO 460                                                          PL3D241
  480 CONTINUE                                                           PL3D242
      NRT = NR(I)                                                        PL3D243
      DO 520  J=1,NRT                                                    PL3D244
         M = J + 1                                                       PL3D245
  484    WRITE(NO,485) I,J,T(I,M-1),IT                                   PL3D246
  485    FORMAT(3X,'SOURCE ',I2,', RATE ',I2,' STARTS AT',F8.1,A3,/,     PL3D247
     1   3X,'ENTER MASS RATE AND ENDING TIME ',/,'   ?,?')               PL3D248
         READ(NI,495,ERR=484) Q(I,M),T(I,M)                              PL3D249
  495    FORMAT(2F10.0)                                                  PL3D250
  500    IF(T(I,M).GT.T(I,M-1)) GO TO 510                                PL3D251
  504    WRITE(NO,505)                                                   PL3D252
  505    FORMAT(3X,'ENDING TIME MUST BE GREATER THAN STARTING TIME '     PL3D253
     1   ' -- REENTER',/,'   ?')                                         PL3D254
         READ(NI,95,ERR=504) T(I,M)                                      PL3D255
         GO TO 500                                                       PL3D256
  510    CONTINUE                                                        PL3D257
  520 CONTINUE                                                           PL3D258
      GO TO 540                                                          PL3D259
  530 WRITE(NO,535) I                                                    PL3D260
  535 FORMAT(3X,'ENTER STEADY-STATE MASS RATE',I2,/,'   ?')              PL3D261
      READ(NI,95,ERR=530) Q(I,1)                                         PL3D262
      NR(I) = 0                                                          PL3D263
  540 CONTINUE                                                           PL3D264
      IF(IEDIT.EQ.2.AND.JFLOW.EQ.1.AND.TF.LE.1.0E-06) GO TO 720          PL3D265
  550 GO TO (560,3000),IEDIT                                             PL3D266
C                                                                        PL3D267
C     COORDINATES OF THE OBSERVATION POINTS                              PL3D268
  560 WRITE(NO,565) IL                                                   PL3D269
  565 FORMAT(3X,'ENTER XIRST, XLAST, DELTAX  (',A2,')' ,/,'   ?,?,?')    PL3D270
      READ(NI,575,ERR=560) XF(1),XL(1),DEL(1)                            PL3D271
  575 FORMAT(3F10.0)                                                     PL3D272
      DEL(1) = ABS(DEL(1))                                               PL3D273
      IF(DEL(1).LE.1.0E-06) XL(1)=XF(1)                                  PL3D274
      GO TO (580,3000),KNTL                                              PL3D275
C                                                                        PL3D276
  580 WRITE(NO,585) IL                                                   PL3D277
  585 FORMAT(3X,'ENTER YFIRST, YLAST, DELTAY  (',A2,')',/,'   ?,?,?')    PL3D278
      READ(NI,575,ERR=580) XF(2),XL(2),DEL(2)                            PL3D279
      DEL(2) = ABS(DEL(2))                                               PL3D280
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      IF(DEL(2).LE.1.0E-06) XL(2)=XF(2)                                  PL3D281
      GO TO (590,3000),KNTL                                              PL3D282
C                                                                        PL3D283
  590 WRITE(NO,595) IL                                                   PL3D284
  595 FORMAT(3X,'ENTER ZFIRST, ZLAST, DELTAZ  (',A2,')',/,'   ?,?,?')    PL3D285
      READ(NI,575,ERR=590) XF(3),XL(3),DEL(3)                            PL3D286
      DEL(3) = ABS(DEL(3))                                               PL3D287
  600 IF(XF(3).GE.0.0.AND.XF(3).LE.ST.AND.DEL(3).LE.1.0E-06) GO TO 620   PL3D288
      IF(XF(3).GE.0.0.AND.XF(3).LE.ST) GO TO 610                         PL3D289
  604 WRITE(NO,605) ST,IL                                                PL3D290
  605 FORMAT(3X,'ZFIRST MUST BE GREATER THAN OR EQUAL TO ZERO AND',/,    PL3D291
     13X,' LESS THAN OR EQUAL TO SATURATED THICKNESS (',F10.4,A3,')',/,  PL3D292
     2' -- REENTER',/,'   ?')                                            PL3D293
      READ(NI,95,ERR=604) XF(3)                                          PL3D294
      GO TO 600                                                          PL3D295
  610 IF(XL(3).GE.0.0.AND.XL(3).LE.ST) GO TO 630                         PL3D296
  614 WRITE(NO,615) ST,IL                                                PL3D297
  615 FORMAT(3X,'ZLAST MUST BE GREATER THAN OR EQUAL TO ZERO AND',/,     PL3D298
     13X,' LESS THAN OR EQUAL TO SATURATED THICKNESS (',F10.4,A3,')',/,  PL3D299
     23X,' -- REENTER',/,'   ?')                                         PL3D300
      READ(NI,95,ERR=614) XL(3)                                          PL3D301
      GO TO 630                                                          PL3D302
  620 XL(3) = XF(3)                                                      PL3D303
  630 GO TO (640,3000), IEDIT                                            PL3D304
C                                                                        PL3D305
C     PLANE FOR SECTIONING AQUIFER                                       PL3D306
  640 WRITE(NO,645)                                                      PL3D307
  645 FORMAT(3X,'ENTER PLANE FOR SECTIONING AQUIFER (XY,XZ,OR YZ)',      PL3D308
     1/,'   ?')                                                         PL3D309
  650 READ(NI,45) ISEC                                                   PL3D310
      DO 660 LSEC=1,3                                                    PL3D311
         IF(ISEC.EQ.KSEC(LSEC)) GO TO 670                                PL3D312
  660 CONTINUE                                                           PL3D313
      WRITE(NO,665)                                                      PL3D314
  665 FORMAT(3X,'INVALID SECTION -- REENTER',/,'   ?')                   PL3D315
      GO TO 650                                                          PL3D316
  670 GO TO (680,690,700), LSEC                                          PL3D317
  680 KHAR1 = KAR3                                                       PL3D318
      KHAR2 = KAR1                                                       PL3D319
      KHAR3 = KAR2                                                       PL3D320
      GO TO 710                                                          PL3D321
  690 KHAR1 = KAR2                                                       PL3D322
      KHAR2 = KAR1                                                       PL3D323
      KHAR3 = KAR3                                                       PL3D324
      GO TO 710                                                          PL3D325
  700 KHAR1 = KAR1                                                       PL3D326
      KHAR2 = KAR2                                                       PL3D327
      KHAR3 = KAR3                                                       PL3D328
  710 GO TO (720,3000),IEDIT                                             PL3D329
C                                                                        PL3D330
C     OBSERVATION TIMES                                                  PL3D331
  720 IF(JFLOW.EQ.2) GO TO 770                                           PL3D332
  724 WRITE(NO,725) IT                                                   PL3D333
  725 FORMAT(3X,'ENTER TFIRST, TLAST, DELTAT  (',A2,')',/,'   ?,?,?')    PL3D334
  730 READ(NI,575,ERR=724) TF,TL,DELT                                    PL3D335
      DELT = ABS(DELT)                                                   PL3D336
  740 IF(TF.GT.0.0.AND.DELT.LE.1.0E-06) GO TO 760                        PL3D337
      IF(TF.GT.0.0) GO TO 750                                            PL3D338
  744 WRITE(NO,745)                                                      PL3D339
  745 FORMAT(3X,'TFIRST MUST BE GREATER THAN ZERO -- REENTER',/,'   ?')  PL3D340
      READ(NI,95,ERR=744) TF                                             PL3D341
      GO TO 740                                                          PL3D342
  750 IF(TL.GT.0.0) GO TO 770                                            PL3D343
  754 WRITE(NO,755)                                                      PL3D344
  755 FORMAT(3X,'TLAST MUST BE GREATER THAN ZERO -- REENTER',/,'   ?')   PL3D345
      READ(NI,95,ERR=754) TL                                             PL3D346
      GO TO 750                                                          PL3D347
  760 TL = TF                                                            PL3D348
  770 GO TO (1000,780),IEDIT                                             PL3D349
  780 IF(JFLOW.EQ.2) WRITE(NO,785)                                       PL3D350
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  785 FORMAT(3X,'TIME IS NOT A PARAMETER IN STEADY-STATE SOLUTION')      PL3D351
      GO TO 3000                                                        PL3D352
C                                                                       PL3D353
C                                                                       PL3D354
C     LIST PROBLEM DEFINITION                                           PL3D355
 1000 WRITE(NO,1005) NPAGE,(TITLE(I),I=1,30)                            PL3D356
 1005 FORMAT(1H1,/,3X,'PLUME3D',/,3X,'VERSION 2.02',                    PL3D357
     1/,3X,'PAGE ',I3,///,3X,30A2,///)                                  PL3D358
      NPAGE = NPAGE + 1                                                 PL3D359
      IF(ST.LE.0.9E32) WRITE(NO,1015) IL,ST                             PL3D360
 1015 FORMAT(1HO,2X,'SATURATED THICKNESS, (',A2,')  ',26X,F10.4)        PL3D361
      WRITE(NO,1025) IL,IT,V,IL,IT,DX,IL,IT,DY,IL,IT,DZ,P               PL3D362
 1025 FORMAT(3X,'SEEPAGE VELOCITY, (',A2,'/',A2,') ',27X,F10.4,/,       PL3D363
     13X,'X DISPERSION COEFFICIENT (',A2,'**2/',A2,')    ',15X,F10.4,/,  PL3D364
     23X,'Y DISPERSION COEFFICIENT (',A2,'**2/',A2,')    ',15X,F10.4,/,  PL3D365
     33X,'Z DISPERSION COEFFICIENT (',A2,'**2/',A2,')    ',15X,F10.4,/,  PL3D366
     43X,'POROSITY ',44X,F10.4)                                         PL3D367
      WRITE(NO,1035) RD,IT,DECAY                                        PL3D368
 1035 FORMAT(//,3X,'RETARDATION COEFFICIENT',30X,F10.4,/,               PL3D369
     13X,'FIRST ORDER DECAY CONSTANT (1/',A2,')',20X,F10.4)             PL3D370
      GO TO (1070,1040), JFLOW                                          PL3D371
 1040 WRITE(NO,1045) IL,IL,IL,IM1,IM2,IM3,IL,IT                         PL3D372
 1045 FORMAT(//,3X,'STEADY-STATE SOURCE RATES',//,                      PL3D373
     13X,'SOURCE',6X,'X',11X,'Y',11X,'Z',17X,'RATE',/,                  PL3D374
     25X,'NO',6X,'(',A2,')',8X,'(',A2,')',8X,'(',A2,')',6X,'(',3A2,     PL3D375
     33')(CU ',A2,'/',A2,')',/)                                         PL3D376
      DO 1060 I=1,NS                                                    PL3D377
         WRITE(NO,1055) I,XS(I),YS(I),ZS(I),Q(I,1)                      PL3D378
 1055    FORMAT(5X,I2,F10.2,2X,F10.2,2X,F10.2,6X,F16.4)                 PL3D379
 1060 CONTINUE                                                          PL3D380
      GO TO 1110                                                        PL3D381
 1070 WRITE(NO,1075) IM1,IM2,IM3,IL,IT,IT,IL,IL,IL                      PL3D382
 1075 FORMAT(//,3X,'SOURCE/RATE SCHEDULE  (',3A2,')(CU ',A2,'/',A2,     PL3D383
     1')'//,15X,'SOURCE',13X,'RATE',4X,'MASS',8X,'TIME (',A2,')',      PL3D384
     2/,3X,'NO   X (',A2,')   Y (',A2,')   Z (',A2,')   NO',5X,'RATE',  PL3D385
     35X,'START',6X,'END',/)                                            PL3D386
      DO 1100  I=1,NS                                                   PL3D387
      WRITE(NO,1085) I,XS(I),YS(I),ZS(I)                                PL3D388
 1085 FORMAT(/,3X,I2,3F9.2)                                             PL3D389
      NRT = NR(I)                                                       PL3D390
      DO 1100  J=1,NRT                                                  PL3D391
         M = J + 1                                                      PL3D392
         WRITE(NO,1095) J,Q(I,M),T(I,M-1),T(I,M)                        PL3D393
 1095    FORMAT(34X,I2,F12.2,2F9.2)                                     PL3D394
 1100 CONTINUE                                                          PL3D395
 1110 WRITE(NO,1115) IL,IT,XF(1),XL(1),DEL(1),XF(2),XL(2),DEL(2),       PL3D396
     1 XF(3),XL(3),DEL(3)                                               PL3D397
 1115 FORMAT(//,3X,'OBSERVATION POINTS (',A2,'), AND TIMES (',A2,')',//, PL3D398
     15X,'XFIRST =',F10.2,5X,'XLAST =',F10.2,5X,'DELX =',F10.4,/,       PL3D399
     25X,'YFIRST =',F10.2,5X,'YLAST =',F10.2,5X,'DELY =',F10.4,/,       PL3D400
     35X,'ZFIRST =',F10.2,5X,'ZLAST =',F10.2,5X,'DELZ =',F10.4)         PL3D401
      IF(JFLOW.EQ.1) WRITE(NO,1125) TF,TL,DELT                          PL3D402
 1125 FORMAT(/,5X,'TFIRST =',F10.2,5X,'TLAST =',F10.2,5X,'DELT =',F10.4) PL3D403
      WRITE(NO,1135) KSEC(LSEC)                                         PL3D404
 1135 FORMAT(//,3X,'AQUIFER SECTIONED IN ',A2,' PLANE')                 PL3D405
      GO TO 3000                                                        PL3D406
C                                                                       PL3D407
C                                                                       PL3D408
C                                                                       PL3D409
C ***** SECTION II -- NUMERICAL EVALUATION OF CONCENTRATION AT          PL3D410
C                     SPECIFIED GRID COORDINATES                        PL3D411
C                                                                       PL3D412
C                                                                       PL3D413
C     NUMBER OF OBSERVATION POINTS IN EACH COORDINATE DIRECTION         PL3D414
 2000 CONTINUE                                                          PL3D415
      DO 2020 L=1,3                                                     PL3D416
         NP(L) = 1                                                      PL3D417
         DEL(L) = ABS(DEL(L))                                           PL3D418
         IF(DEL(L).LE.1.0E-03) GO TO 2020                               PL3D419
         DIF = XL(L) - XF(L)                                            PL3D420
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          IF(ABS(DIF).LE.1.0E-03) GO TO 2020                          PL3D421
          IF(DIF.LE.0.0) DEL(L)=-DEL(L)                               PL3D422
          NPTS = ABS(DIF/DEL(L))                                      PL3D423
          REM = DIF - DEL(L)*FLOAT(NPTS)                              PL3D424
          NPTS = NPTS + 1                                             PL3D425
          NP(L) = NPTS                                                PL3D426
          IF(ABS(REM).LT.1.0E-03) GO TO 2020                          PL3D427
          NP(L) = NP(L) + 1                                           PL3D428
 2020 CONTINUE                                                        PL3D429
      GO TO (2040,2060,2080),LSEC                                     PL3D430
 2040 MAXSC = NP(3)                                                   PL3D431
      MAXRW = NP(2)                                                   PL3D432
      MAXCL = NP(1)                                                   PL3D433
      GO TO 2100                                                      PL3D434
 2060 MAXSC = NP(2)                                                   PL3D435
      MAXRW = NP(3)                                                   PL3D436
      MAXCL = NP(1)                                                   PL3D437
      GO TO 2100                                                      PL3D438
 2080 MAXSC = NP(1)                                                   PL3D439
      MAXRW = NP(3)                                                   PL3D440
      MAXCL = NP(2)                                                   PL3D441
 2100 CONTINUE                                                        PL3D442
C                                                                     PL3D443
C     TIME COORDINATES                                                PL3D444
      NTIME = 1                                                       PL3D445
      IF(DELT.LE.1.0E-06) GO TO 2110                                  PL3D446
      NTIME = ABS(TL-TF)/DELT + 1.0                                   PL3D447
      IF(TF.GT.TL) DELT=-DELT                                         PL3D448
 2110 TSOL = TF                                                       PL3D449
      MTIME = NTIME                                                   PL3D450
C                                                                     PL3D451
C                                                                     PL3D452
      DAMK = DX*DECAY*RD/(V*V)                                        PL3D453
      ALPHA=SQRT(1.0+4.0*DAMK)                                        PL3D454
      PE=V/DX                                                         PL3D455
      BETA = DX/DY                                                    PL3D456
      GAMMA = DX/DZ                                                   PL3D457
      LAMBDA = 1.0/(25.132741*P*SQRT(DY*DZ))                          PL3D458
C                                                                     PL3D459
C                                                                     PL3D460
      DO 2660  NT=1,NTIME                                             PL3D461
C                                                                     PL3D462
      NSEC = 1                                                        PL3D463
 2120 LPRT = 1                                                        PL3D464
      LP = 1                                                          PL3D465
      NCFLG = 1                                                       PL3D466
 2140 NROW1 = 1                                                       PL3D467
      NROW2 = MAXROW                                                  PL3D468
 2160 IF(NROW2.GT.MAXRW) NROW2=MAXRW                                  PL3D469
      DO 2580  NROW=NROW1,NROW2                                       PL3D470
      GO TO (2180,2220,2200),NCFLG                                    PL3D471
 2180 NCOL1 = 1                                                       PL3D472
      NCOL2 = MAXCOL                                                  PL3D473
 2200 IF(NCOL2.GT.MAXCL) NCOL2=MAXCL                                  PL3D474
      NCOL = MAXCOL                                                   PL3D475
      IF(NCOL2.EQ.MAXCL) NCOL=NCOL2-NCOL1+1                           PL3D476
 2220 GO TO (2240,2260,2280), LSEC                                    PL3D477
 2240 IX1 = NCOL1                                                     PL3D478
      IX2 = NCOL2                                                     PL3D479
      JY1 = NROW                                                      PL3D480
      JY2 = NROW                                                      PL3D481
      KZ1 = NSEC                                                      PL3D482
      KZ2 = NSEC                                                      PL3D483
      GO TO 2290                                                      PL3D484
 2260 IX1 = NCOL1                                                     PL3D485
      IX2 = NCOL2                                                     PL3D486
      JY1 = NSEC                                                      PL3D487
      JY2 = NSEC                                                      PL3D488
      KZ1 = NROW                                                      PL3D489
      KZ2 = NROW                                                      PL3D490
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      GO TO 2290                                                        PL3D491
 2280 IX1 = NSEC                                                        PL3D492
      IX2 = NSEC                                                        PL3D493
      JY1 = NCOL1                                                       PL3D494
      JY2 = NCOL2                                                       PL3D495
      KZ1 = NROW                                                        PL3D496
      KZ2 = NROW                                                        PL3D497
 2290 CONTINUE                                                          PL3D498
C                                                                       PL3D499
      DO 2300  L=1,MAXCOL                                               PL3D500
         CON(L) = 0.0                                                   PL3D501
 2300 CONTINUE                                                          PL3D502
C                                                                       PL3D503
      DO 2440   N=1,NS                                                  PL3D504
         D(1) = ST - ZS(N)                                              PL3D505
         IF(ST.GE.0.9E32) D(1)=0.0                                      PL3D506
         D(2) = ZS(N)                                                   PL3D507
         D(3) = D(1)                                                    PL3D508
         COEF = 1.0                                                     PL3D509
         IF(D(1).LT.1.0E-03.OR.D(2).LT.1.0E-03) COEF=2.0                PL3D510
         DO 2440  I=IX1,IX2                                             PL3D511
            X = XF(1) + FLOAT(I-1)*DEL(1)                               PL3D512
            IF(I.EQ.NP(1)) X=XL(1)                                      PL3D513
            XXS = X - XS(N)                                             PL3D514
            PEX = PE*XXS                                                PL3D515
            DO 2440  J=JY1,JY2                                          PL3D516
               Y = XF(2) + FLOAT(J-1)*DEL(2)                            PL3D517
               IF(J.EQ.NP(2)) Y=XL(2)                                   PL3D518
               YYS = Y - YS(N)                                          PL3D519
               PEY = PE*YYS                                             PL3D520
               DO 2430  K=KZ1,KZ2                                       PL3D521
                  L = I-IX1 + J-JY1 + K-KZ1 + 1                         PL3D522
                  IF(CON(L).LT.0.0) GO TO 2430                          PL3D523
                  Z = XF(3) + FLOAT(K-1)*DEL(3)                         PL3D524
                  IF(K.EQ.NP(3)) Z=XL(3)                                PL3D525
                  ZM = Z - ZS(N)                                        PL3D526
                  IF(ABS(XXS).LT.1.0.AND.ABS(YYS).LT.1.0.AND.           PL3D527
     1             ABS(ZM).LT.1.0) GO TO 2330                           PL3D528
                  PEZ = PE*ZM                                           PL3D529
                  CALL SOL3D(C,PEX,PEY,PEZ,TSOL,N,NR(N))               PL3D530
                  CXYZT = COEF*C                                        PL3D531
                  IF(IMAGE.EQ.1) GO TO 2325                             PL3D532
C                                                                       PL3D533
                  DO 2320  LM=1,2                                       PL3D534
                     ZM = ((-1.0)**(LM+1))*ZM                           PL3D535
                     IF(D(LM).LT.1.0E-03) GO TO 2320                    PL3D536
                     ZIMAGE = 2.0*D(LM) - ZM                            PL3D537
                     PEZ = PE*ZIMAGE                                    PL3D538
                     CALL SOL3D(C,PEX,PEY,PEZ,TSOL,N,NR(N))             PL3D539
                     CXYZT = CXYZT + COEF*C                             PL3D540
                     DO 2310  IM = 1,IMAGE                              PL3D541
                        ZIMAGE = (2.0*D(LM)+ZM) + 2.0*FLOAT(IM)*D(LM+1) PL3D542
     1                          + FLOAT(2*IM-2)*D(LM)                   PL3D543
                        PEZ = PE*ZIMAGE                                 PL3D544
                        CALL SOL3D(C,PEX,PEY,PEZ,TSOL,N,NR(N))          PL3D545
                        IF(C.LT.1.0E-06) GO TO 2312                     PL3D546
                        CXYZT = CXYZT + COEF*C                          PL3D547
 2310                CONTINUE                                           PL3D548
 2312                CONTINUE                                           PL3D549
                     DO 2314  IM=1,IMAGE                                PL3D550
                        ZIMAGE = (2.0*D(LM)-ZM) + 2.0*FLOAT(IM)*D(LM+1) PL3D551
     1                          + FLOAT(2*IM)*D(LM)                     PL3D552
                        PEZ = PE*ZIMAGE                                 PL3D553
                        CALL SOL3D(C,PEX,PEY,PEZ,TSOL,N,NR(N))          PL3D554
                        IF(C.LT.1.0E-06) GO TO 2320                     PL3D555
                        CXYZT = CXYZT + COEF*C                          PL3D556
 2314                CONTINUE                                           PL3D557
                     WRITE(NO,2315) MAXING,X,Y,Z                        PL3D558
 2315                FORMAT(3X,'****** WARNING -- SOLUTION DID NOT',    PL3D559
     1               ' CONVERGE USING'./,9X,I2,' IMAGE WELLS AT X =',   PL3D560
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     2              F10.4,'  Y =',F10.4,'  Z =',F10.4)                    PL3D561
 2320          CONTINUE                                                   PL3D562
 2325          CON(L) = CON(L) + CXYZT                                    PL3D563
               GO TO 2340                                                 PL3D564
 2330          CON(L) = -9.9999                                           PL3D565
 2340          GO TO (2360,2380,2400), LSEC                               PL3D566
 2360          SEC = Z                                                    PL3D567
               ROW = Y                                                    PL3D568
               COL(L) = X                                                 PL3D569
               GO TO 2420                                                 PL3D570
 2380          SEC = Y                                                    PL3D571
               ROW = Z                                                    PL3D572
               COL(L) = X                                                 PL3D573
               GO TO 2420                                                 PL3D574
 2400          SEC = X                                                    PL3D575
               ROW = Z                                                    PL3D576
               COL(L) = Y                                                 PL3D577
 2420          CONTINUE                                                   PL3D578
 2430       CONTINUE                                                      PL3D579
 2440  CONTINUE                                                          PL3D580
C                                                                        PL3D581
C      PRINT CONCENTRATION DISTRIBUTION                                  PL3D582
       GO TO (2460,2560), LPRT                                           PL3D583
 2460  WRITE(NO,1005) NPAGE, (TITLE(I),I=1,30)                           PL3D584
       NPAGE = NPAGE + 1                                                 PL3D585
       IF(JFLOW.EQ.2) GO TO 2500                                         PL3D586
       WRITE(NO,2465) TSOL,IT,IM1,IM2,IM3,KHAR1,SEC,IL,                  PL3D587
      1(LBL(LP,L),L=1,6),KHAR2,IL                                        PL3D588
 2465  FORMAT(13X,'CONCENTRATION DISTRIBUTION AT ',F10.2,                PL3D589
      11X,A2,' (',3A2,') ',//,13X,A1,' =',F10.2,1X,A2,3X,6A2,//,         PL3D590
      2' =',/,'    = ',A1,'(',A2,')')                                    PL3D591
       GO TO 2520                                                        PL3D592
 2500  WRITE(NO,2505) IM1,IM2,IM3,KHAR1,SEC,IL,(LBL(LP,L),L=1,6),       PL3D593
      1KHAR2,IL                                                          PL3D594
 2505  FORMAT(13X,'CONCENTRATION DISTRIBUTION AT STEADY STATE',          PL3D595
      1' (',3A2,') ',//13X,A1,' =',F10.2,1X,A2,3X,6A2,//,                PL3D596
      2' =',/,'    = ',A1,'(',A2,')')                                    PL3D597
 2520  CONTINUE                                                          PL3D598
       WRITE(NO,2525) (COL(L),L=1,NCOL)                                  PL3D599
 2525  FORMAT('      =',4X,7F10.2)                                       PL3D600
       WRITE(NO,2545) KHAR3,IL                                           PL3D601
 2545  FORMAT(1X,A1,'(',A2,') =',/,9X,'=')                               PL3D602
C                                                                        PL3D603
 2560  WRITE(NO,2565) ROW,(CON(L),L=1,NCOL)                              PL3D604
 2565  FORMAT(2X,F8.2,7F10.4)                                            PL3D605
       LPRT = 2                                                          PL3D606
 2580  CONTINUE                                                          PL3D607
       IF(NROW2.EQ.MAXRW) GO TO 2600                                     PL3D608
       NROW1 = NROW1 + MAXROW                                            PL3D609
       NROW2 = NROW2 + MAXROW                                            PL3D610
       LPRT = 1                                                          PL3D611
       LP = 2                                                            PL3D612
       NCFLG = 2                                                         PL3D613
       GO TO 2160                                                        PL3D614
 2600  IF(NCOL2.EQ.MAXCL) GO TO 2620                                     PL3D615
       NCOL1 = NCOL1 + MAXCOL                                            PL3D616
       NCOL2 = NCOL2 + MAXCOL                                            PL3D617
       LPRT = 1                                                          PL3D618
       LP = 2                                                            PL3D619
       NCFLG = 3                                                         PL3D620
       GO TO 2140                                                        PL3D621
 2620  IF(NSEC.EQ.MAXSC) GO TO 2640                                      PL3D622
       NSEC = NSEC + 1                                                   PL3D623
       GO TO 2120                                                        PL3D624
 2640  CONTINUE                                                          PL3D625
       TSOL = TSOL + DELT                                                PL3D626
       IF(NT.EQ.MTIME) TSOL=TL                                           PL3D627
 2660  CONTINUE                                                          PL3D628
C                                                                        PL3D629
C                                                                        PL3D630
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C ***** SECTION III -- PROBLEM REDEFINITION AND CONTROL OF EXECUTION      PL3D631
C                                                                         PL3D632
C                                                                         PL3D633
 3000 CONTINUE                                                            PL3D634
      IF(IEDIT.EQ.2) GO TO 3010                                           PL3D635
      WRITE(NO,3705)                                                      PL3D636
      IEDIT = 2                                                           PL3D637
 3010 KNTL = 2                                                            PL3D638
      WRITE(NO,3015)                                                      PL3D639
 3015 FORMAT(//,3X,'ENTER NEXT COMMAND',/,'   ?')                         PL3D640
 3020 READ(NI,3025) NEXT                                                  PL3D641
 3025 FORMAT(A2)                                                          PL3D642
C                                                                         PL3D643
      DO 3030  I=1,21                                                     PL3D644
         IF(NEXT.EQ.IC(I)) GO TO 3040                                     PL3D645
 3030 CONTINUE                                                            PL3D646
      WRITE(NO,3035)                                                      PL3D647
 3035 FORMAT(3X,'ERROR IN LAST COMMAND -- REENTER',/,'   ?')              PL3D648
      GO TO 3020                                                          PL3D649
 3040 GO TO (10,80,120,160,190,220,250,280,310,320,560,580,590,640,       PL3D650
     1       720,1000,2000,3050,3060,3700,4000),I                        PL3D651
C                                                                         PL3D652
C     NEW SET OF X, Y, AND Z OBSERVATIONS                                 PL3D653
 3050 KNTL = 1                                                            PL3D654
      GO TO 560                                                           PL3D655
C                                                                         PL3D656
C                                                                         PL3D657
C                                                                         PL3D658
C     NEW SOURCE/RATE SCHEDULE                                            PL3D659
 3060 WRITE(NO,3065)                                                      PL3D660
 3065 FORMAT(3X,'ADD(A),DELETE(D),MODIFY(M) A SOURCE OR RETURN(R)'        PL3D661
     1' TO EDIT ?')                                                       PL3D662
 3070 READ(NI,3075) ISK                                                   PL3D663
 3075 FORMAT(A1)                                                          PL3D664
      DO 3080  I=1,4                                                      PL3D665
         IF(ISK.EQ.IS(I)) GO TO 3090                                      PL3D666
 3080 CONTINUE                                                            PL3D667
      WRITE(NO,3085)                                                      PL3D668
 3085 FORMAT(3X,'ERROR IN SELECTION -- REENTER ?')                        PL3D669
      GO TO 3070                                                          PL3D670
 3090 GO TO (3000,3100,3450,3490),I                                       PL3D671
C                                                                         PL3D672
C     MODIFY SOURCE                                                       PL3D673
C                                                                         PL3D674
 3100 WRITE(NO,3105) NS                                                   PL3D675
 3105 FORMAT(3X,I2,' SOURCES IN CURRENT SCHEDULE',/,                      PL3D676
     13X,'ENTER SOURCE TO MODIFY',/,'   ?')                              PL3D677
      READ(NI,465,ERR=3100) FDUM                                          PL3D678
      JS=FDUM                                                             PL3D679
      IF(JS.GT.0.AND.JS.LE.NS) GO TO 3220                                 PL3D680
      WRITE(NO,3215) JS                                                   PL3D681
 3215 FORMAT(3X,'SOURCE',I4,' NOT IN SCHEDULE')                           PL3D682
      GO TO 3060                                                          PL3D683
 3220 GO TO (3230,3260),JFLOW                                             PL3D684
 3230 WRITE(NO,3235) JS,XS(JS),IL,YS(JS),IL,ZS(JS),IL,IT,                 PL3D685
     1IM1,IM2,IM3,IL,IT                                                   PL3D686
 3235 FORMAT(3X,'SOURCE ',I2,':   X =',F8.2,A3,',  Y =',F8.2,A3,         PL3D687
     1',  Z =',F8.2,A3,//,3X,'RATE',7X,'MASS RATE',14X,'TIME (',         PL3D688
     2A2,')',/,4X,'NO',3X,'(',3A2,')(CU ',A2,'/',A2,')',                  PL3D689
     38X,'START',7X,'END',/)                                              PL3D690
      NRT = NR(JS)                                                        PL3D691
      DO 3250  J=1,NRT                                                    PL3D692
         M = J + 1                                                        PL3D693
         WRITE(NO,3245) J,Q(JS,M),T(JS,M-1),T(JS,M)                       PL3D694
 3245    FORMAT(4X,I2,5X,F14.2,7X,F8.3,3X,F8.2)                           PL3D695
 3250 CONTINUE                                                            PL3D696
      GO TO 3270                                                          PL3D697
 3260 WRITE(NO,3265) JS,XS(JS),IL,YS(JS),IL,ZS(JS),IL,Q(JS,1),            PL3D698
     1IM1,IM2,IM3,IL,IT                                                   PL3D699
 3265 FORMAT(3X,'SOURCE ',I2,':   X =',F8.2,A3,',  Y =',F8.2,A3,         PL3D700
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     1',  Z=',F8.2,A3,/,3X,'STEADY-STATE MASS RATE =',F16.4,          PL3D701
     2' (',3A3,')(CU ',A2,'/',A2,')',/)                               PL3D702
 3270 WRITE(NO,3275)                                                  PL3D703
 3275 FORMAT(3X,'CHANGE COORDINATES (Y/N)?')                          PL3D704
      READ(NI,3075) JC                                                PL3D705
      IF(JC.NE.IY) GO TO 3290                                         PL3D706
 3276 WRITE(NO,415) JS,IL                                             PL3D707
      READ(NI,425,ERR=3276) XS(JS),YS(JS),ZS(JS)                      PL3D708
 3280 IF(ZS(JS).GE.O.O.AND.ZS(JS).LE.ST) GO TO 3290                   PL3D709
 3284 WRITE(NO,435) ST,IL                                             PL3D710
      READ(NI,95,ERR=3284) ZS(JS)                                     PL3D711
      GO TO 3280                                                      PL3D712
 3290 GO TO (3300,3430),JFLOW                                         PL3D713
C                                                                     PL3D714
C     TRANSIENT SOURCES                                               PL3D715
 3300 WRITE(NO,3305) JS                                               PL3D716
 3305 FORMAT(3X,'MODIFY RATE SCHEDULE FOR SOURCE',I3,' (Y/N) ?')      PL3D717
      READ(NI,3075) JY                                                PL3D718
      IF(JY.NE.IY) GO TO 3060                                         PL3D719
 3310 WRITE(NO,3315)                                                  PL3D720
 3315 FORMAT(3X,'ENTER RATE TO BE CHANGED',/,                         PL3D721
     13X,'(ENTER O TO CHANGE ALL RATES)',/,'   ?')                   PL3D722
      READ(NI,465,ERR=3310) FDUM                                      PL3D723
      JR=FDUM                                                         PL3D724
      IF(JR.LE.O) GO TO 3350                                          PL3D725
      IF(JR.LE.NR(JS)) GO TO 3330                                     PL3D726
      WRITE(NO,3325) JR                                               PL3D727
 3325 FORMAT(3X,'RATE ',I2,' NOT IN CURRENT SCHEDULE')                PL3D728
      GO TO 3300                                                      PL3D729
 3330 WRITE(NO,3335) JS,JR,T(JS,JR),T(JS,JR+1),IT                     PL3D730
 3335 FORMAT(3X,'SOURCE ',I2,', RATE ',I2,' STARTS AT',F8.2,          PL3D731
     1' AND ENDS AT',F8.2,A3,/,3X,'ENTER NEW MASS RATE',/,'   ?')     PL3D732
      M = JR + 1                                                      PL3D733
      READ(NI,3345,ERR=3330) Q(JS,M)                                  PL3D734
 3345 FORMAT(F10.O)                                                   PL3D735
      GO TO 3300                                                      PL3D736
C                                                                     PL3D737
 3350 NRT = NR(JS)                                                    PL3D738
      DO 3360  J=1,NRT                                                PL3D739
         M = J + 1                                                    PL3D740
         Q(JS,M) = O.O                                                PL3D741
         T(JS,M) = O.O                                                PL3D742
 3360 CONTINUE                                                        PL3D743
 3370 WRITE(NO,455) JS,MAXRT                                          PL3D744
 3380 READ(NI,465,ERR=3370) FDUM                                      PL3D745
      NR(JS)=FDUM                                                     PL3D746
      IF(NR(JS).GT.O.AND.NR(JS).LE.MAXRT) GO TO 3390                  PL3D747
      WRITE(NO,475) MAXRT                                             PL3D748
      GO TO 3380                                                      PL3D749
 3390 CONTINUE                                                        PL3D750
      NRT = NR(JS)                                                    PL3D751
      DO  3420  J=1,NRT                                               PL3D752
         M = J + 1                                                    PL3D753
 3394    WRITE(NO,485) JS,J,T(JS,M-1),IT                              PL3D754
         READ(NI,495,ERR=3394) Q(JS,M),T(JS,M)                        PL3D755
 3400    IF(T(JS,M).GT.T(JS,M-1)) GO TO 3410                          PL3D756
 3404    WRITE(NO,505)                                                PL3D757
         READ(NI,95,ERR=3404) T(JS,M)                                 PL3D758
         GO TO 3400                                                   PL3D759
 3410    CONTINUE                                                     PL3D760
 3420 CONTINUE                                                        PL3D761
      GO TO 3060                                                      PL3D762
C                                                                     PL3D763
C     STEADY-STATE SOURCES                                            PL3D764
 3430 WRITE(NO,3435) JS                                               PL3D765
 3435 FORMAT(3X,'CHANGE STEADY-STATE RATE FOR SOURCE ',I2,' (Y/N) ?')  PL3D766
      READ(NI,3075) JC                                                PL3D767
      IF(JC.NE.IY) GO TO 3060                                         PL3D768
 3444 WRITE(NO,3445) JS                                               PL3D769
 3445 FORMAT(3X,'ENTER NEW STEADY-STATE MASS RATE FOR SOURCE ',I2,/,  PL3D770
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     1'   ?')                                                          PL3D771
      READ(NI,3345,ERR=3444) Q(JS,1)                                   PL3D772
      GO TO 3060                                                       PL3D773
C                                                                      PL3D774
C     ADD A NEW SOURCE                                                 PL3D775
C                                                                      PL3D776
 3450 NS = NS + 1                                                      PL3D777
      JS = NS                                                          PL3D778
 3454 WRITE(NO,415) JS,IL                                              PL3D779
      READ(NI,425,ERR=3454) XS(JS),YS(JS),ZS(JS)                       PL3D780
 3460 IF(ZS(JS).GE.O.O.AND.ZS(JS).LE.ST) GO TO 3470                    PL3D781
 3464 WRITE(NO,435) ST,IL                                              PL3D782
      READ(NI,95,ERR=3464) ZS(JS)                                      PL3D783
      GO TO 3460                                                       PL3D784
 3470 GO TO (3370,3480),JFLOW                                          PL3D785
C                                                                      PL3D786
C     STEADY-STATE SOURCES                                             PL3D787
 3480 WRITE(NO,3485) JS                                                PL3D788
 3485 FORMAT(3X,'ENTER STEADY-STATE MASS RATE FOR SOURCE ',I2,         PL3D789
     1/,'   ?')                                                        PL3D790
      READ(NI,3345,ERR=3480) Q(JS,1)                                   PL3D791
      NR(JS) = O                                                       PL3D792
      GO TO 3060                                                       PL3D793
C                                                                      PL3D794
C     DELETE A SOURCE                                                  PL3D795
C                                                                      PL3D796
 3490 IF(NS.GT.1) GO TO 3500                                           PL3D797
      WRITE(NO,3495)                                                   PL3D798
 3495 FORMAT(3X,'ONLY ONE SOURCE IN SCHEDULE -- CAN NOT DELETE',/)     PL3D799
      GO TO 3060                                                       PL3D800
 3500 WRITE(NO,3505) IL,IL,IL                                          PL3D801
 3505 FORMAT(3X,'SOURCE',6X,'X (',A2,')',3X,'Y (',A2,')',3X,           PL3D802
     1'Z (',A2,')',/)                                                 PL3D803
      DO 3520  I=1,NS                                                  PL3D804
         WRITE(NO,3515) I,XS(I),YS(I),ZS(I)                            PL3D805
 3515    FORMAT(5X,I2,3X,F8.2,3X,F8.2,3X,F8.2)                         PL3D806
 3520 CONTINUE                                                         PL3D807
 3530 WRITE(NO,3535)                                                   PL3D808
 3535 FORMAT(3X,'ENTER SOURCE TO DELETE',/,                            PL3D809
     13X,'(ENTER O TO CANCEL)',/,'   ?')                              PL3D810
      READ(NI,465,ERR=3530) FDUM                                       PL3D811
      JS=FDUM                                                          PL3D812
      IF(JS.LE.O) GO TO 3060                                           PL3D813
      IF(JS.LE.NS) GO TO 3550                                          PL3D814
      WRITE(NO,3545) JS                                                PL3D815
 3545 FORMAT(3X,'SOURCE ',I2,' NOT IN CURRENT SCHEDULE')               PL3D816
      GO TO 3530                                                       PL3D817
 3550 WRITE(NO,3555) JS                                                PL3D818
 3555 FORMAT(3X,'DELETE SOURCE ',I2,' (Y/N)?')                         PL3D819
      READ(NI,3075) JC                                                 PL3D820
      IF(JC.NE.IY) GO TO 3530                                          PL3D821
      NSD = NS - 1                                                     PL3D822
      GO TO (3560,3590),JFLOW                                          PL3D823
C                                                                      PL3D824
C     TRANSIENT SOURCES                                                PL3D825
 3560 IF(JS.EQ.NS) GO TO 3575                                          PL3D826
      DO  3570  J=JS,NSD                                               PL3D827
         XS(J) = XS(J+1)                                               PL3D828
         YS(J) = YS(J+1)                                               PL3D829
         ZS(J) = ZS(J+1)                                               PL3D830
         NR(J) = NR(J+1)                                               PL3D831
         NRT = NR(J)                                                   PL3D832
            DO 3570  K=1,NRT                                           PL3D833
            M = K + 1                                                  PL3D834
            Q(J,M) = Q(J+1,M)                                          PL3D835
            T(J,M) = T(J+1,M)                                          PL3D836
 3570 CONTINUE                                                         PL3D837
 3575 NRT = NR(NS)                                                     PL3D838
      DO 3580  K=1,NRT                                                 PL3D839
         M = K + 1                                                     PL3D840
```

```
         Q(NS,M) = 0.0                                                    PL3D841
         T(NS,M) = 0.0                                                    PL3D842
 3580 CONTINUE                                                            PL3D843
      NR(NS) = 0                                                          PL3D844
      NS = NSD                                                            PL3D845
      GO TO 3060                                                          PL3D846
C                                                                         PL3D847
C     STEADY-STATE SOURCES                                                PL3D848
 3590 IF(JS.EQ.NS) GO TO 3605                                             PL3D849
      DO 3600  J=JS,NSD                                                   PL3D850
         Q(J,1) = Q(J+1,1)                                                PL3D851
         XS(J) = XS(J+1)                                                  PL3D852
         YS(J) = YS(J+1)                                                  PL3D853
         ZS(J) = ZS(J+1)                                                  PL3D854
 3600 CONTINUE                                                            PL3D855
 3605 Q(NS,1) = 0.0                                                       PL3D856
      NS = NSD                                                            PL3D857
      GO TO 3060                                                          PL3D858
C                                                                         PL3D859
C                                                                         PL3D860
C     MENU OF EDIT COMMANDS FOR PLUME3D VERSION 2.02                      PL3D861
 3700 WRITE(NO,3705)                                                      PL3D862
 3705 FORMAT(1H1,/,3X,'MENU OF EDIT COMMANDS',//,                         PL3D863
     13X,'SATURATED THICKNESS        ST      OBSERVATION POINTS   OB',/, PL3D864
     23X,'POROSITY                   PO      X COORDINATES        XC',/, PL3D865
     33X,'SEEPAGE VELOCITY           VX      Y COORDINATES        YC',/, PL3D866
     43X,'RETARDATION COEFFICIENT    RD      Z COORDINATES        ZC',/, PL3D867
     53X,'X DISPERSION COEFFICIENT   DX      OBSERVATION TIMES    TC',/, PL3D868
     53X,'Y DISPERSION COEFFICIENT   DY      AQUIFER SECTIONING   AS',/, PL3D869
     63X,'Z DISPERSION COEFFICIENT   DZ      NEW PROBLEM          NP',/, PL3D870
     73X,'DECAY CONSTANT             DE      MENU OF COMMANDS     MU',/, PL3D871
     83X,'SOURCE/RATE SCHEDULE       RT      LIST INPUT DATA      LI',/, PL3D872
     93X,'CHANGE SOLUTION/SOURCES    CS      RUN CALCULATIONS     RN',/, PL3D873
     13X,'                                   DONE                 DN')  PL3D874
      GO TO 3000                                                          PL3D875
C                                                                         PL3D876
 4000 STOP                                                                PL3D877
      END                                                                 PL3D878
```

```
      FUNCTION ERFCLG(Z)                                                    EFLG001
C     RATIONAL APPROXIMATION OF THE COMPLIMENTARY ERROR FUNCTION            EFLG002
C     SEE SECTION 7.1 OF ABRAMOWITZ AND STEGUN (1966)                       EFLG003
C     THE FOLLOWING IDENTITIES ARE USED TO HANDLE NEGATIVE ARGUMENTS        EFLG004
C        ERFC( Z) = 1 - ERF(Z)                                              EFLG005
C        ERF (-Z) = -ERF (Z)                                                EFLG006
C                                                                           EFLG007
      REAL*8 COEFLG,DERFC,DI,FX,TERMI,TERMO,SUM,X                           EFLG008
      COMMON/IO/NI,NO                                                       EFLG009
C                                                                           EFLG010
      X = ABS(Z)                                                            EFLG011
      IF (X.GT.3.0D00) GO TO 50                                             EFLG012
C                                                                           EFLG013
C     FOR X<3 A RATIONAL APPROXIMATION OF THE COMPLIMENTARY ERROR           EFLG014
C     FUNCTION IS USED.                                                     EFLG015
C                                                                           EFLG016
      DERFC = 1.0D00/((1.0D00 + 7.05230784D-02*X + 4.22820123D-02*(X**2)    EFLG017
     1                 + 9.2705272D-03*(X**3) + 1.520143D-04*(X**4)         EFLG018
     2                 + 2.76572D-04*(X**5) + 4.306380-05*(X**6))**16)      EFLG019
      ERFCLG = DLOG(DERFC)                                                  EFLG020
      GO TO 100                                                             EFLG021
C                                                                           EFLG022
C     FOR X>3 AN ASYMPTOTCI EXPANSION OF THE COMPLIMENTARY ERROR            EFLG023
C     FUNCTION IS USED.                                                     EFLG024
   50 COEFLG = X*X + DLOG(X) + 0.57236494D00                                EFLG025
      FX = 2.0D00*X*X                                                       EFLG026
      SUM = 1.0D00                                                          EFLG027
      TERMO = 1.0D00                                                        EFLG028
      DO 60 I=2,50                                                          EFLG029
      DI = I                                                                EFLG030
      TERMI = -TERMO*(2.0D00*DI - 3.0D00)/FX                                EFLG031
      IF(DABS(TERMI).GT.DABS(TERMO)) GO TO 70                               EFLG032
      SUM = SUM + TERMI                                                     EFLG033
      TEST = TERMI/SUM                                                      EFLG034
      IF(ABS(TEST).LT.1.0E-16) GO TO 70                                     EFLG035
      TERMO = TERMI                                                         EFLG036
   60 CONTINUE                                                              EFLG037
      WRITE(NO,65)                                                          EFLG038
   65 FORMAT(6X,'*** WARNING -- ASYMPTOTIC EXPANSION FOR ERFC DID NOT',     EFLG039
     1'                    CONVERGE WITH 50 TERMS IN THE SUMMATION')       EFLG040
   70 SUM = DLOG(SUM) - COEFLG                                              EFLG041
      ERFCLG = SUM                                                          EFLG042
  100 CONTINUE                                                              EFLG043
C                                                                           EFLG044
C     FOR Z<0, ERFC(-Z) = 2-ERFC(Z)                                         EFLG045
      IF(Z.LT.0.0) GO TO 200                                                EFLG046
      RETURN                                                                EFLG047
  200 ERFC = 2.0 - EXP(ERFCLG)                                              EFLG048
      ERFCLG = ALOG(ERFC)                                                   EFLG049
      RETURN                                                                EFLG050
      END                                                                   EFLG051
```

```
      SUBROUTINE SOL3D(C,PEX,PEY,PEZ,TSOL,N,NR)                          SOL3001
C     NUMERICAL EVALUATION OF ANALYTICAL SOLUTION                        SOL3002
C                                                                        SOL3003
      REAL LAMBDA                                                        SOL3004
      COMMON/RATE/Q(10,12),T(10,12)                                      SOL3005
      COMMON/PHYPRO/ALPHA,BETA,GAMMA,DX,LAMBDA,PE,RD,V                   SOL3006
C                                                                        SOL3007
      PEXYZ = SQRT(PEX*PEX + BETA*(PEY*PEY) + GAMMA*(PEZ*PEZ))           SOL3008
      R = PEXYZ/PE                                                       SOL3009
      PEL = PEXYZ*ALPHA                                                  SOL3010
      PEX12 = PEX/2.0                                                    SOL3011
      PEL12 = PEL/2.0                                                    SOL3012
      COEF = LAMBDA/R                                                    SOL3013
      MT = NR + 1 .                                                      SOL3014
      IF(MT.GT.1) GO TO 10                                               SOL3015
C                                                                        SOL3016
C     STEADY-STATE SOLUTION                                              SOL3017
C                                                                        SOL3018
      S = Q(N,1)                                                         SOL3019
      IF(CLG.LT.-72.0) CLG = -72.0                                       SOL3020
      IF(CLG.GT. 72.0) CLG =  72.0                                       SOL3021
      C = 2.0*EXP(CLG)                                                   SOL3022
      GO TO 50                                                           SOL3023
C                                                                        SOL3024
C     TRANSIENT SOLUTION                                                 SOL3025
   10 C = 0.0                                                            SOL3026
      IF(T(N,MT).LT.TSOL) MT=MT+1                                        SOL3027
      DO 40  K=2,MT                                                      SOL3028
         IF(T(N,K-1).GT.TSOL) GO TO 50                                   SOL3029
         S = Q(N,K) - Q(N,K-1)                                           SOL3030
         SGN = 1.0                                                       SOL3031
         IF(S.LT.0.0) SGN = -1.0                                         SOL3032
         SA = ABS(S)                                                     SOL3033
         IF(SA.LT.1.0E-3) GO TO 40                                       SOL3034
         COEFLG = ALOG(COEF*SA)                                          SOL3035
         PINJ = V*V*(TSOL-T(N,K-1))/(DX*RD)                              SOL3036
         PINJL = PINJ*ALPHA*ALPHA                                        SOL3037
         TAU = PINJ/(PEXYZ*PEXYZ)                                        SOL3038
         Z1 = 0.5/SQRT(TAU)                                              SOL3039
         Z2 = 0.5*SQRT(PINJL)                                            SOL3040
         Z = Z1 + Z2                                                     SOL3041
         T1 = ERFCLG(Z)                                                  SOL3042
         C1LG = T1 + PEL12 + PEX12 + COEFLG                              SOL3043
         Z = Z1 - Z2                                                     SOL3044
         T2 = ERFCLG(Z)                                                  SOL3045
         C2LG = T2 - PEL12 + PEX12 + COEFLG                              SOL3046
         IF(C1LG.GT. 72.0) C1LG= 72.0                                    SOL3047
         IF(C1LG.LT.-72.0) C1LG=-72.0                                    SOL3048
         IF(C2LG.GT. 72.0) C2LG= 72.0                                    SOL3049
         IF(C2LG.LT.-72.0) C2LG=-72.0                                    SOL3050
         CK = EXP(C1LG) + EXP(C2LG)                                      SOL3051
         C = C + SGN*CK                                                  SOL3052
   40 CONTINUE                                                           SOL3053
   50 RETURN                                                             SOL3054
      END                                                                SOL3055
```

APPENDIX C

Solute Transport in Uniform Ground-Water Flow
Mathematical Development

SOLUTE TRANSPORT IN UNIFORM GROUND-WATER FLOW

MATHEMATICAL DEVELOPMENT

Analytical solute-transport models are highly idealized mathematical approximations of complex physical phenomena. However, if applied properly this class of models represents a useful tool for analyzing the transport and fate of substances under both field and laboratory conditions.

This mathematical derivation of a governing differential equation for solute transport in uniform ground-water flow is developed in an effort to show the relationship between physical processes in the aquifer and terms in the mathematical model. In this context, the "mathematical manipulations" of the derivation are not as important as the simplifying assumptions which are required to obtain a mathematical description of the problem.

The resulting differential equation serves as the basis of many analytical solute-transport models. Solutions of the differential equation which would provide the actual concentration distributions in time and space are not addressed in this document. However, the simplifying assumptions which are incorporated in the formulation of the mathematical model must be considered in both the application of the model and in the interpretation of modeling results, regardless of how the model is solved.

Derivation or the Governing Differential Equation.

Consider a differential element of homogeneous aquifer as shown in Figure 1. A material balance for any tracer can be written as

$$\begin{matrix}\text{Rate of} \\ \text{Mass In}\end{matrix} - \begin{matrix}\text{Rate of} \\ \text{Mass Out}\end{matrix} + \begin{matrix}\text{Rate of Mass} \\ \text{Generation}\end{matrix} = \begin{matrix}\text{Rate of Mass} \\ \text{Accumulation}\end{matrix} \qquad (1)$$

Each of the terms in this expression are developed in the following paragraphs.

Rate of Mass In:

The tracer can enter the differential control volume by two mechanisms. The first is convection, or bulk flow, of tracer in solution. This mechanism can be expressed as

$$Q_x C|_x + Q_y C|_y + Q_z C|_z$$

where Q_x, Q_y and Q_z are the volumetric flow rates in the x, y and z directions, respectively, and C is the concentration of tracer in the fluid.

Tracer can also enter the control volume by molecular diffusion and hydrodynamic dispersion. For the time being these processes can be combined as a "dispersion flux" term, q", with dimensions of mass input per unit time per unit area. Thus,

$$q''_x|_x \, \Delta y \Delta z + q''_y|_y \, \Delta z \Delta z + q''_z|_z \, \Delta z \Delta y$$

represents the rate of mass entering the control volume by "dispersion."

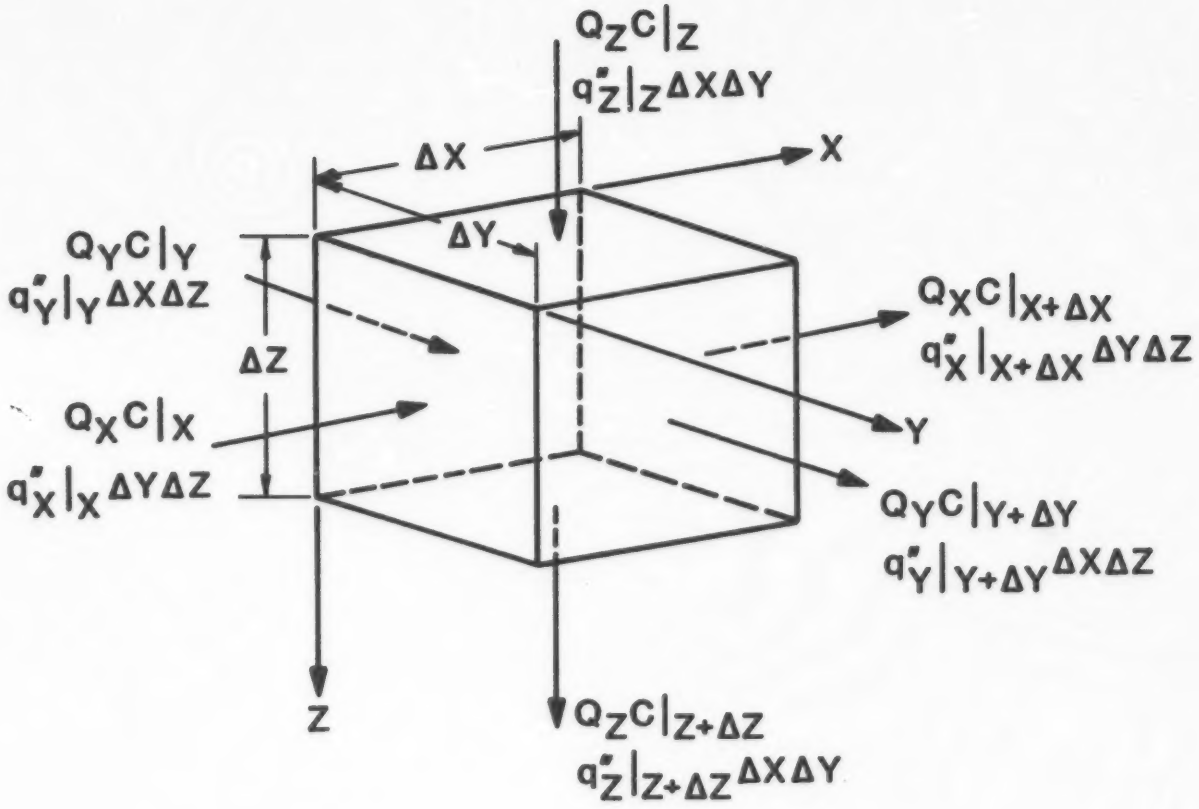

Figure 1 - Differential Control Volume for Mass Balance

Rate of Mass Out:

Tracer can leave the control volume by the same mechanisms described for the input terms. Thus, the rates of mass leaving by convection and dispersion can be written as

$$Q_x C|_{x+\Delta x} + Q_y C|_{y+\Delta y} + Q_z C|_{z+\Delta z}$$

and

$$q''_x|_{x+\Delta x}\ \Delta y \Delta z + q''_y|_{y+\Delta y}\ \Delta x \Delta z + q''_z|_{z+\Delta z}\ \Delta x \Delta y$$

respectively

Rate of Mass Generation:

Tracer can be generated (or degraded) in the control volume by physical, chemical, and/or biological reaction. A general relationship can be written as

$$r_T\ \Delta x \Delta y \Delta z$$

where r_T is the total, or overall, rate of generation per unit volume of aquifer.

Rate of Mass Accumulation:

The total rate of mass accumulated in the control volume during a differential period of time is

$$\frac{C_T|_{t+\Delta t} - C_T|_t}{\Delta t} (\Delta x \Delta y \Delta z)$$

where C_T is the total mass of trace per unit volume of aquifer.

Substituting the expressions for the various terms into Equation 1, and dividing by $(\Delta x \Delta y \Delta z)$ yields

$$- \frac{Q_x C|_{x+\Delta x} - Q_x C|_x}{\Delta x \ (\Delta y \Delta z)} - \frac{Q_y C|_{y+\Delta y} - Q_y C|_y}{\Delta y \ (\Delta x \Delta z)} - \frac{Q_z C|_{z+\Delta z} - Q_z C|_z}{\Delta z \ (\Delta x \Delta y)}$$

$$- \frac{q''_x|_{x+\Delta x} - q''_x|_x}{\Delta x} - \frac{q''_y|_{y+\Delta y} - q''_y|_y}{\Delta y} - \frac{q''_z|_{z+\Delta z} - q''_z|_z}{\Delta z} + r_T$$

$$= \frac{(C_T)_{t+\Delta t} - (C_T)_t}{\Delta t} \qquad (2)$$

Now

$$V_x \equiv \frac{Q_x}{\Delta y \Delta z} \qquad (3a)$$

$$V_y \equiv \frac{Q_y}{\Delta z \Delta x} \qquad (3b)$$

and

$$V_z \equiv \frac{Q_z}{\Delta x \Delta y} \qquad (3c)$$

where V is the superficial, or Darcy, velocity. Taking the limit of Equation 2 as Δx, Δy, Δz, and Δt go to zero yields

$$- \frac{\partial(V_x C)}{\partial_x} - \frac{\partial(V_y C)}{\partial_y} - \frac{\partial(V_z C)}{\partial_z} - \frac{\partial q''_x}{\partial_x} - \frac{\partial q''_y}{\partial_y} - \frac{\partial q''_z}{\partial_z} + r_T = \frac{\partial C_T}{\partial_t} \qquad (4)$$

Equation 4 is the differential mass balance for a tracer in a porous medium.

If a Fickian model is assumed for the dispersion flux, these fluxes can be expressed in terms of concentration gradients. Neglecting surface diffusion of tracer which may be adsorbed on the solid matrix,

$$q''_n = - D_n \frac{\partial(\theta C)}{\partial_n} \qquad (5)$$

where n is the coordinate direction, D_n is a dispersion coefficient, and θ is the effective porosity or fractional void volume. The concentration gradient in the liquid phase is the driving force for mass tranport by dispersion. The porosity has been included since the mass balance has been formulated for a unit volume of aquifer, which includes the solid matrix as well as the fluid-filled pores. Substituting an expression of the form of Equation 5 for each of the dispersion flux terms in Equation 4 yields

$$\frac{\partial C_T}{\partial_x} + \frac{\partial(V_x C)}{\partial_x} + \frac{\partial(V_y C)}{\partial_y} + \frac{\partial(V_z C)}{\partial_z}$$

$$= \frac{\partial}{\partial_x} D_x \frac{\partial(\theta C)}{\partial_x} + \frac{\partial}{\partial_y} D_y \frac{\partial(\theta C)}{\partial_y} + \frac{\partial}{\partial_z} D_z \frac{\partial(\theta C)}{\partial_z} + r_T \qquad (6)$$

Two assumptions will be made at this point. The first is that the aquifer is homogeneous, which implies that the porosity and dispersion coefficients are not functions of position. The second assumption is that the ground-water flow is uniform and directed along the x-axis, i.e., V_x = constant and $V_y = V_z = 0$. With these assumptions, Equation 6 reduces to

$$\frac{\partial C_T}{\partial t} + V_x \frac{\partial C}{\partial x} = \theta D_x \frac{\partial^2 C}{\partial x^2} + \theta D_y \frac{\partial^2 C}{\partial y^2} + \theta D_z \frac{\partial^2 C}{\partial z^2} + r_T \qquad (7)$$

This equation is a statement of conservation of tracer in homogeneous aquifer with uniform ground-water flow. The accumulation and reaction terms are developed in the following paragraphs.

Accumulation. In general, the total mass of tracer per unit volume of aquifer, C_T, can be distributed as dissolved solute in the fluid-filled pore volume and as adsorbed solute on the solid matrix, or

$$\frac{\text{mass of tracer}}{\text{bulk volume}} = \frac{\text{mass of tracer}}{\text{volume of solution}} \frac{\text{volume of solution}}{\text{bulk volume}}$$

$$+ \frac{\text{mass of tracer}}{\text{mass of solids}} \frac{\text{mass of solids}}{\text{bulk volume}} \qquad (8)$$

Equation 8 can be written in terms of aquifer properties as

$$C_T = \theta C + \rho_B C_s \qquad (9)$$

where ρ_B is the bulk density of the solid matrix and C_s is the adsorbed mass concentration (mass of tracer per unit mass of solids). For a homogeneous aquifer, the accumulation term in Equation 7 can be written as

$$\frac{\partial C_T}{\partial t} = \theta \frac{\partial C}{\partial t} + \rho_B \frac{\partial C_s}{\partial t} \qquad (10)$$

In general, the concentration of absorbed solute, C_s, is a function of the concentration of solute in solution, C, and

$$\frac{\partial C_s}{\partial t} = \frac{dC_s}{dC} \frac{\partial C}{\partial t} \tag{11}$$

For a linear adsorption isotherm,

$$C_s = K_d C \tag{12}$$

or

$$\frac{dC_s}{dC} = K_d \tag{13}$$

where K_d is a constant commonly referred to as an adsorption, or distribution, coefficient. The rate of accumulation of tracer per unit volume of aquifer can be expressed in terms of the concentration in solution by combining Equations 10, 11 and 13 as follows:

$$\frac{\partial C_T}{\partial t} = \theta \frac{\partial C}{\partial t} + \rho_B K_d \frac{\partial C}{\partial t} \tag{14}$$

The coefficients of $\partial C/\partial t$ are often combined as

$$R_d \equiv 1 + \frac{K_d \rho_B}{\theta} \tag{15}$$

where R_d is referred to as a "retardation coefficient." Rewriting Equation 14 as

$$\frac{\partial C_T}{\partial t} = \theta R_d \frac{\partial C}{\partial t} \tag{14}$$

or

$$\frac{\partial C_T}{\partial t} = \theta \frac{\partial C}{\partial (t/R_d)} \tag{15}$$

gives some insight into the effect of adsorption on accumulation of tracer in the homogeneous aquifer. The apparent effect is a distortion or "retardation" of the time dimension.

Reaction. Only first-order reactions will be considered in formulating the rate of reaction term, r_T, in Equation 7. The kinetic models for reaction of tracer in solution and adsorbed tracer are

$$\frac{\partial C}{\partial t} = - \lambda_f C \tag{16}$$

and

$$\frac{\partial C_s}{\partial t} = - \lambda_s C_s \tag{17}$$

where λ_f and λ_s are the fluid phase and solid phase rate coefficients, respectively. Equations 16 and 17 have been written for degradation of tracer, i.e. negative generation. The overall rate of reaction can be written as

$$r_T \equiv \frac{\partial C_T}{\partial t} = - \theta \lambda_f C - \rho_B \lambda_s C_s \tag{18}$$

Including the linear adsorption isotherm developed above

$$C_s = K_d C$$

and

$$r_T = - \theta \left(\lambda_f + \frac{\rho_B K_d}{\theta} \lambda_s\right) C \qquad (19)$$

or

$$r_T = - \theta \lambda_T C \qquad (20)$$

where λ_T is an apparent overall first-order rate constant defined as

$$\lambda_T \equiv \lambda_f + \frac{\rho_B K_d}{\theta} \lambda_s \qquad (21)$$

For radioactive decay, the rate of reaction is usually expressed in terms of the "half-life" or the time required for the concentration to be reduced to one-half of the initial concentration, $t_{1/2}$. Integrating Equation 16 or 17 from $t = 0$ to $t = t_{1/2}$

$$\int_{C_o}^{C_o/2} \frac{dC}{C} = - \lambda \int_{o}^{t_{1/2}} dt$$

or

$$\ln C \Big|_{C_o}^{C_o/2} = - \lambda t_{1/2}$$

Solving for the rate constant,

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

which is an expression for evaluating a first-order rate constant from the half-life of the reaction.

In the case of radioactive decay, the rate constants are independent of the phase in which the reaction is occuring, and

$$\lambda = \lambda_f = \lambda_s$$

Equation 21 can then be written as

$$\lambda_T = R_d \lambda$$

and Equation 19 becomes

$$r_T = - \theta R_d \lambda C \qquad (22)$$

Equation 22 applies to all cases where the first-order reaction rate constants are the same for both fluid and solid phase reactions.

Differential Equation in Terms of Fluid Phase Concentrations.

The differential mass balance for a tracer in a homogeneous aquifer with uniform ground-water flow, Equation 7, can be written in terms of fluid-phase concentrations by incorporating Equation 14 for linear adsorption and Equation 19 for first-order reactions. Making these substitutions and rearranging yields

$$R_d \frac{\partial C}{\partial t} + V^* \frac{\partial C}{\partial x} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - \lambda_T C \quad (23)$$

where V* is the average interstitial, or pore, velocity defined as

$$V^* \equiv \frac{V}{\theta} \quad (24)$$

Integration of Equation 23 with appropriate initial and boundary conditions yields the temporal and spacial distribution of a tracer in a homogeneous aquifer with uniform ground-water flow.

Closed-form analytical solutions to Equation 23 can be obtained by making a change of variables. Let

$$\tau = t/R_d \quad (25)$$

and

$$X = x - V^*\tau \quad (26)$$

Now, $C = C(x,y,z,t)$, and holding y and z constant

$$\left(\frac{\partial C}{\partial \tau}\right)_X = \left(\frac{\partial C}{\partial X}\right)_\tau \left(\frac{\partial X}{\partial \tau}\right)_X + \left(\frac{\partial C}{\partial \tau}\right)_x \left(\frac{\partial \tau}{\partial \tau}\right)_X \quad (27)$$

Also,

$$\left(\frac{\partial x}{\partial \tau}\right)_X = \left(\frac{\partial X}{\partial \tau}\right)_X + V^* \left(\frac{\partial \tau}{\partial \tau}\right)_X \quad (28)$$

Substituting Equation 28 into Equation 27 yields

$$\left(\frac{\partial C}{\partial \tau}\right)_x = \left(\frac{\partial C}{\partial \tau}\right)_X - V^* \left(\frac{\partial C}{\partial x}\right)_\tau \tag{29}$$

For the second derivative term in x,

$$\left(\frac{\partial^2 C}{\partial X^2}\right)_\tau = \frac{\partial}{\partial X}\left(\frac{\partial X}{\partial X}\right)_\tau \left[\left(\frac{\partial C}{\partial X}\right)_\tau \left(\frac{\partial x}{\partial X}\right)_\tau\right] = \frac{\partial^2 C}{\partial x^2} \tag{30}$$

Substituting Equations 25, 29 and 30 into Equation 23 yields

$$\frac{\partial C}{\partial \tau} = D_x \frac{\partial^2 C}{\partial X^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - \lambda_T C \tag{31}$$

which is a special form of the heat conduction equation. Closed-form analytical solutions for this equation are available in the literature for a variety of boundary conditions.

Summary.

Equation 23 provides the basis for many of the analytical solutions for solute transport in uniform ground-water flow. This equation is a mathematical model of complex physical phenomena and incorporates many simplyfying assumptions which are required to obtain a solution to the problem. Assumptions incorporated in the formulation of the differential equation are also present in the solution and must be considered in interpreting any numerical results.

The assumption that the aquifer is homogeneous is seldom satisfied in practical field problems. Also, the use of an equilibrium adsorption isotherm implies that adsorption of solute on the solid matrix is both reversible and

instantaneous. Although these assumptions are seldom met in either field or laboratory problems, the approximations to the physical system may be reasonable for initial estimates of concentration distributions.

Solutions to Equation 23 with a continuous source of tracer are often encountered in the literature. The assumption of a uniform velocity in the x-direction makes no provision for the effects of a high volumetric source rate on the flow-field in the region of the source. For most problems, this assumption is probably reasonable at moderate distances from either sources or sinks of fluid, or on a regional basis.

The use of a Fickian model for the dispersive flux is probably the most frequently misinterpreted or incorrectly applied portion of the mathematical model. Hydrodynamic dispersion is an observed effect of one or more physical phenomena which are difficult to define and cannot be measured. A discussion of the topic is beyond the scope of this brief treatise. However, the mathematical formulation of the problem as developed in this paper treats dispersion as a potential flow problem. Analogous mechanism are conduction in heat transfer and molecular diffusion in mass transfer. Thus, the model does not distinguish between hydrodynamic dispersion in the direction of ground-water flow or opposite to the direction of ground-water flow. The model is a statement of conservation of tracer, and solutions of the governing differential equation can lead to higher concentrations upgradient of sources (and thus lower concentrations downgradient) than would be observed in practice.

Analytical solutions to the solute transport equation present viable and valuable alternatives for analyzing fairly complex problems, even with the simplifying assumptions which have been incorporated. Interpretation of the results of an analytical solution to the problem must be based on an understanding of both the physical system and the mathematical model.